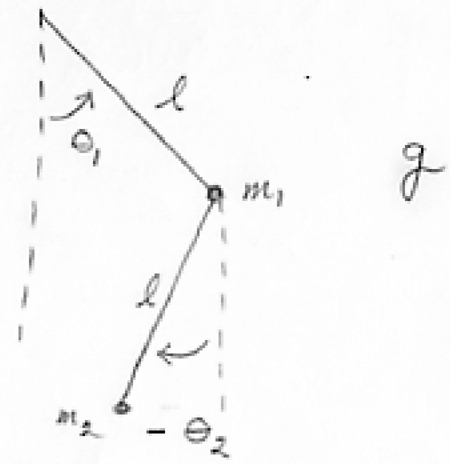
<u>Goldstein</u> <u>6.A</u>

Dragt 92

See solution to Goldstein 1.22 for $\mathcal{L}$. (Dragt 33)



There we found

$$\mathcal{L} = \frac{1}{2}\left\{ (m_1+m_2)\,\ell^2\dot{\theta}_1^2 + m_2\ell^2\dot{\theta}_2^2 + 2m_2\ell^2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2) + (m_1+m_2)\,g\,\ell\cos\theta_1 + m_2 g\ell\cos\theta_2 \right\}.$$

The Equilibrium position is $\theta_1 = \theta_2 = 0$. For small oscillations,

$$T = \frac{1}{2}(m_1+m_2)\,\ell^2\dot{\theta}_1^2 + m_2\ell^2\dot{\theta}_2^2 + m_2\ell^2\dot{\theta}_1\dot{\theta}_2 + O(\epsilon^3)$$

$$V = -m_1 g\ell - 2m_2 g\ell + \frac{1}{2}(m_1+m_2)\,g\ell\,\theta_1^2 + m_2 g\ell\,\theta_2^2 + O(\epsilon^3)$$

$$\therefore\ \mathcal{L}_{eff} = \frac{1}{2}(m_1+m_2)\,\ell^2\dot{\theta}_1^2 + \frac{1}{2}m_2\ell^2\dot{\theta}_2^2 + m_2\ell^2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}(m_1+m_2)\,g\ell\,\theta_1^2 - \frac{1}{2}m_2 g\ell\,\theta_2^2.$$