

# Goldstein 6.4

Solution to Problem set 14

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See solution to Goldstein

1.22 For  $\mathcal{L}$ . (Draft 33)

See homework problem 7.13 for  $\mathcal{L}$

There we found

$$\mathcal{L} = \frac{1}{2} \left\{ (m_1 + m_2) \ell^2 \dot{\theta}_1^2 + m_2 \ell^2 \dot{\theta}_2^2 \right.$$

$$\left. + 2 m_2 \ell^2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2) g \ell \cos \theta_1 + m_2 g \ell \cos \theta_2 \right\}$$

The Equilibrium position is  $\theta_1 = \theta_2 = 0$ .

For small oscillations,

$$T = \frac{1}{2} (m_1 + m_2) \ell^2 \dot{\theta}_1^2 + m_2 \ell^2 \dot{\theta}_2^2 + m_2 \ell^2 \dot{\theta}_1 \dot{\theta}_2 + O(\epsilon^3)$$

$$V = -m_1 g \ell - 2 m_2 g \ell + \frac{1}{2} (m_1 + m_2) g \ell \theta_1^2 + m_2 g \ell \theta_2^2 + O(\epsilon^3)$$

$$\therefore \mathcal{L}_{\text{eff}} = \frac{1}{2} (m_1 + m_2) \ell^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \ell^2 \dot{\theta}_2^2 + m_2 \ell^2 \dot{\theta}_1 \dot{\theta}_2 - \frac{1}{2} (m_1 + m_2) g \ell \theta_1^2$$

$$- \frac{1}{2} m_2 g \ell \theta_2^2$$

