Dragt 100 cont. To sum the series use the trig identity

$$\sin\left(\frac{n\pi v}{L}t\right)\sin\left(\frac{n\pi x}{L}\right) = \frac{1}{2}\left\{\cos\left[\left(\frac{n\pi}{L}\right)(x-vt)\right] - \cos\left[\left(\frac{n\pi}{L}\right)(x+vt)\right]\right\}$$

This gives $q(x,t) = f(x-vt) - f(x+vt)$ where

$$f(\xi) = \sum_{1}^{\infty} \frac{2D}{n\pi}\left(\sin\frac{n\pi}{2}\right)(1/2)\cos\left[\left(\frac{n\pi}{L}\right)\xi\right].$$

We see that $f(\xi)$ is even and has period $2L$.

We see that $f(0) = \frac{2D}{\pi}\left(\frac{1}{2}\right)\sum_{1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{2}$

$$= \frac{D}{\pi}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right] = \frac{D}{\pi}\left(\frac{\pi}{4}\right) = D/4.$$

48.31. $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}E_0 = \frac{\pi}{4}$. $[E_0 = 1$ as in 46.1.$]$

We also have the result

SPECIAL FOURIER SERIES AND THEIR GRAPHS

23.7 $f(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & -\pi < x < 0 \end{cases}$

$$\frac{4}{\pi}\left(\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right)$$

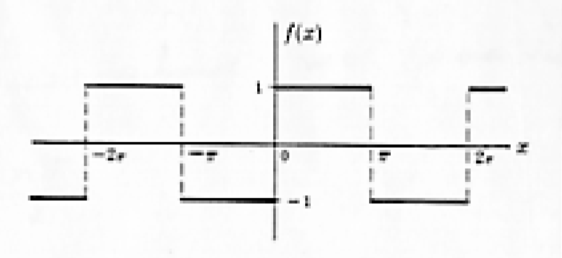

Fig. 23-1