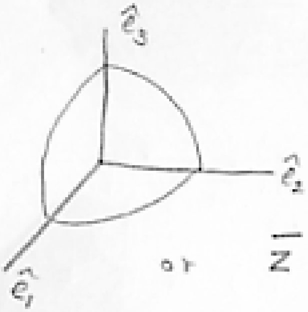
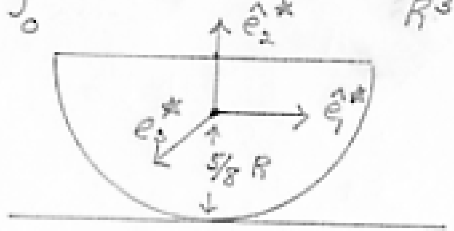
First Locate $\underline{cm}$ of hemisphere by turning it upside down.




Then $\bar{x} = \bar{y} = 0$, $\bar{z} = \dfrac{\int z \rho \, d^3\vec{r}}{\int \rho \, d^3 r} = \dfrac{\rho}{\frac{2}{3}\pi R^3 \rho} \int r\cos\theta \, \sin\theta \, d\theta \, d\phi \, r^2 dr$

or $\bar{z} = \dfrac{3}{2\pi R^3}(2\pi) \int_0^R r^3 dr \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta = \dfrac{3}{R^3} \left. \dfrac{r^4}{4} \right|_0^R \left. \dfrac{1}{2} \sin^2\theta \right|_0^{\pi/2} = \dfrac{3}{8} R$

So, we have following picture:




Next calculate $I_{ij}^{CM}$ using Steiner // axis theorem.

Begin by calculating $I_{ij}^{0}$ where O is center of original sphere. We have the result:

$$I_{ij}^{0} = \int_{\text{hemisphere}} d^3\vec{r} \, \rho(\vec{r}) \left\{ \delta_{ij} \vec{r}^{\,2} - r_i r_j \right\} = 0 \quad \text{for } i \neq j \text{ by symmetry}$$

$$I_{11}^{0} (= I_{33}^{0}) = \int_{\text{hemisphere}} d^3\vec{r} \, \rho(\vec{r}) \left\{ r_2^2 + r_3^2 \right\} = \frac{1}{2} \left. I_{11}^{0} \right|_{\text{whole sphere}} = \frac{1}{2} M \frac{2}{5} R^2 = \frac{1}{5} M R^2$$

$$I_{22}^{0} = \int_{\text{hemisphere}} d^3\vec{r} \, \rho(\vec{r}) \left\{ r_3^2 + r_1^2 \right\} = \frac{1}{2} \left. I_{22}^{0} \right|_{\text{whole sphere}} = \frac{1}{2} M \frac{2}{5} R^2 = \frac{1}{5} M R^2$$