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a) According to the problem, $\hat{e}_1^*$ is along the axis of the disc, and $\hat{e}_f^* = \hat{e}_f$ initially. Therefore, $\boxed{R(t) = e^{\beta J_3} e^{\alpha J_1}.}$

Use $\sum \omega_i^* J_i = R^{-1}\dot{R}$. Then,

$$\dot{R} = \dot{\beta} J_3 e^{\beta J_3} e^{\alpha J_1} + e^{\beta J_3} \dot{\alpha} J_1 e^{\alpha J_1} \text{ and } R^{-1} = e^{-\alpha J_1} e^{-\beta J_3}.$$

$$\therefore R^{-1}\dot{R} = \dot{\beta} e^{-\alpha J_1} J_3 e^{\alpha J_1} + \dot{\alpha} J_1 = \dot{\beta}\{J_3 \cos\alpha + J_2 \sin\alpha\} + \dot{\alpha} J_1.$$

$$\therefore \boxed{\begin{aligned} \omega_1^* &= \dot{\alpha} = \tfrac{d}{dt}(at) = a \\ \omega_2^* &= \dot{\beta}\sin\alpha = \{\sin(at)\}\tfrac{d}{dt}\{bt + ct^2\} = (b+2ct)\sin(at). \\ \omega_3^* &= \dot{\beta}\cos\alpha = (b+2ct)\cos(at) \end{aligned}}$$

b)
$$I_{ij}^* = \begin{pmatrix} \frac{1}{2}MR^2 & 0 & 0 \\ 0 & \left(\frac{MR^2}{4} + \frac{Md^2}{12}\right) & 0 \\ 0 & 0 & \left(\frac{MR^2}{4} + \frac{Md^2}{12}\right) \end{pmatrix} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$