Dragt 8.7

a) By definition, $I_{ij} = \int d^3\vec{r}\, \rho(\vec{r}) \left[\delta_{ij}\, \vec{r}^{\,2} - r_i r_j \right]$.

$\therefore \quad I_{ii} = \int d^3\vec{r}\, \rho(\vec{r}) \left[r_j^2 + r_k^2 \right] \geq 0$ since $\rho \geq 0$.

b) $I_{ii} + I_{jj} = \int d^3\vec{r}\, \rho(\vec{r}) \left[r_j^2 + r_k^2 + r_k^2 + r_i^2 \right]$

$I_{kk} = \int d^3\vec{r}\, \rho(\vec{r}) \left[r_j^2 + r_i^2 \right]$

$\therefore \boxed{I_{ii} + I_{jj} \geq I_{kk}}$ Note that this is the triangular inequality.

c) $I = \begin{pmatrix} 2 & 4 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. $\det(I - \lambda E) = \begin{vmatrix} 2-\lambda & 4 & 0 \\ 4 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)\left\{ (2-\lambda)^2 - 16 \right\} = (3-\lambda)(-2-\lambda)(6-\lambda)$. $\therefore$ eigenvalues are $3, 6, -2$.

The eigenvalues are supposed to be principal moments, and $\lambda = -2$ <u>violates</u> $I_{ii} \geq 0$.