Now use Lagrangian.

$$\mathcal{L} = T - V = \tfrac{1}{2} I \dot{\theta}^2 - V$$

$$\Rightarrow \mathcal{L} = \left[\left(\tfrac{1}{5} M_s + \tfrac{1}{3} M_c \right) R^2 + \frac{(M_s + M_c)}{2} L^2 \right] \dot{\theta}^2$$

$$- (M_s - M_c)\, g L \cos\theta.$$

This gives the equation of motion

$$\left[\left(\tfrac{2}{5} M_s + \tfrac{2}{3} M_c \right) R^2 + (M_s + M_c) L^2 \right] \ddot{\theta}$$

$$+ (M_c - M_s)\, g L \sin\theta = 0$$

so
$$\omega^2 = \frac{(M_c - M_s)\, g L}{\left(\tfrac{2}{5} M_s + \tfrac{2}{3} M_c \right) R^2 + (M_s + M_c) L^2}$$

Computing M_s & M_c in terms of ρ & R $\Rightarrow$

$$\boxed{\omega^2 = \frac{\left(8 - \frac{4\pi}{3} \right) g L}{\left(\frac{8\pi}{15} + \frac{16}{3} \right) R^2 + \left(8 + \frac{4\pi}{3} \right) L^2}}$$

$$\boxed{\tau = 2\pi / \omega}$$