a) $T = T_{\text{of CM}} + T_{\text{Rot}}$. $T_{\text{of CM}} = \frac{1}{2} M \dot{q}^2$. $T_{\text{Rot}} = \frac{1}{2} I \omega^2$.

$I = \frac{1}{2} M R^2$ and $\omega R = \dot{q} \Rightarrow T_{\text{Rot}} = \frac{1}{4} M R^2 \omega^2 = \frac{1}{4} M \dot{q}^2$

$V = \frac{1}{2} k (q-d)^2 + \frac{1}{2} k (q+d)^2 = k q^2 + k d^2$ where d is separation of two ends of springs. Drop the term kd^2 since it has no effect. $L = T - V \Rightarrow$

$$\boxed{L = \tfrac{3}{4} M \dot{q}^2 - k q^2}$$

b) Equilibrium is at $q = 0$, and eqn of motion is

$$\tfrac{3}{2} M \ddot{q} + 2kq = 0 \Rightarrow$$

$$\omega^2 = \frac{2k}{3/2\, M} = \frac{4k}{3M} \quad \text{or} \quad \boxed{\omega = \left(\frac{4k}{3M}\right)^{1/2}}$$