Dragt 83

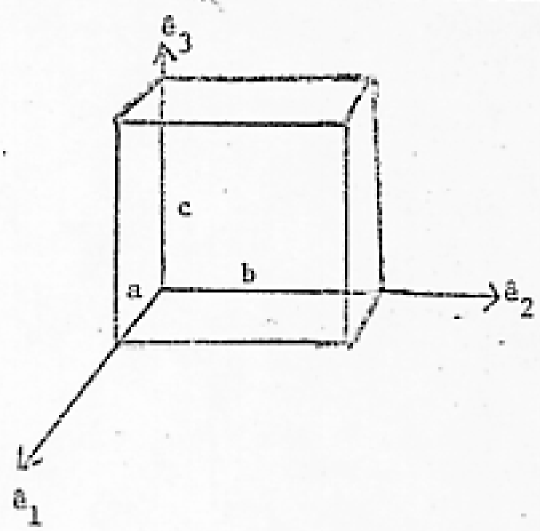
$$I_{12} = -\int \rho\, d^3\vec{r}\; x y$$

$$= -\frac{M}{abc}\, c \int_0^a x\,dx \int_0^b y\,dy$$

$$= -\frac{M}{abc}\, c \left(\frac{a^2}{2}\right)\left(\frac{b^2}{2}\right) = -\frac{M}{4}\, a b$$

Points

14. Give the moment of inertia tensor for the parallelepiped shown below. It has mass M, sides a, b, and c, and the origin is at one corner as shown below.



$$I_{ij} = \int d^3\vec{r}\, \rho \left[\delta_{ij} r^2 - r_i r_j\right]$$

$$I_{11} = \int d^3 r\, \rho \left[r^2 - x^2\right]$$

$$= \int d^3 r\, \rho \left[y^2 + z^2\right]$$

$$= \frac{M}{abc}\left[ac \int_0^b y^2\,dy + ab \int_0^c z^2\,dz\right]$$

$$= \frac{M}{abc}\left[\frac{acb^3}{3} + \frac{abc^3}{3}\right] = \frac{M}{3}(b^2+c^2)$$

6 $$I = \begin{pmatrix} \frac{M}{3}(b^2+c^2) & -\frac{M}{4}ab & -\frac{M}{4}ac \\ -\frac{M}{4}ab & \frac{M}{3}(a^2+c^2) & -\frac{M}{4}bc \\ -\frac{M}{4}ac & -\frac{M}{4}bc & \frac{M}{3}(a^2+b^2) \end{pmatrix}$$