

DLN 1.3.4

$$\dot{V} = k V^{2/3} + V(0) = 0 \Rightarrow V^{-2/3} dV = k dt \Rightarrow$$

$$3 V^{1/3} = k(t - \tau) \Rightarrow V = [k/3(t - \tau)]^3. \text{ Note that}$$

$$\dot{V}(t) \Big|_{t=\tau} = 3 [k/3(t - \tau)]^2 (k/3) \Big|_{t=\tau} = 0. \text{ Also, } V(0) = 0$$

is also a solution. Therefore these solutions can be joined at  $t = \tau$  to produce the solution

$$V(t) = 0, \quad -\infty \leq t \leq \tau \quad \tau \geq 0$$

$$= [k/3(t - \tau)]^3, \quad t \geq \tau.$$

which obviously satisfies the initial condition  $V(0) = 0$

The lack of uniqueness is caused by  $f = kV^{2/3}$  and  $\frac{\partial f}{\partial V}$  being singular at  $V = 0 = V(0)$ .