DLN 1.3.1

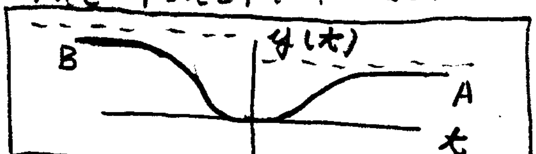
It is easily verified using L'Hopital's Rule that the function $y(t)$ defined by

$$y(t) = A e^{-(1/t)^2}, \quad t \neq 0$$
$$= 0, \quad t = 0$$

is C^∞ and obeys $d^n y/dt^n\big|_{t=0} = 0$ for all n. By differentiation

$$\dot{y} = Ae^{-(1/t)^2}(2/t^3) \Rightarrow \boxed{t^3 \dot{y} = 2y \quad \forall t} \quad ★$$

It follows that the function defined by



$$\boxed{\begin{aligned} y(t) &= A e^{-(1/t^2)}, \quad t > 0 \\ &= 0, \quad t = 0 \\ &= B e^{-(1/t^2)}, \quad t < 0 \end{aligned}}$$

is also $\underline{\underline{C^\infty}}$ and satisfies

the differential equation ★

with the initial condition $y(0) = 0$ for all A, B.

Writing ★ in the form $\dot{y} = f(y,t)$ we see that $f = 2t^{-3}y$ + ∴ $\frac{\partial f}{\partial t}$ is singular at $t=0$. Thus the conditions for theorem 1 are violated.