

To show that  $M^N X^{a^*} = X^b$ ,

observe that when the Bernoulli map  $\mathcal{B}$  acts on  $a^*$  we get

$$\mathcal{B} a_1 \cdot a_2 a_3 \cdots a_N b_1 b_2 b_3 \cdots = a_2 \cdot a_3 \cdots a_N b_1 b_2 \cdots$$

$$\mathcal{B}^2 a_1 \cdot a_2 a_3 \cdots a_N b_1 b_2 b_3 \cdots = a_3 \cdot a_4 \cdots a_N b_1 b_2 \cdots$$

etc.

$$\Rightarrow \mathcal{B}^N a_1 \cdot a_2 a_3 \cdots a_N b_1 b_2 b_3 \cdots = b_1 \cdot b_2 b_3 \cdots = b.$$