Let us compare $a = a_1 . a_2 a_3 \cdots a_N a_{N+1} \cdots$ and

$$a^* = a_1 . a_2 a_3 \cdots a_N b_1 b_2 b_3 \cdots$$

We have

$$a = a_1 + \frac{a_2}{2} + \frac{a_3}{2^2} + \cdots + \frac{a_N}{2^{N-1}} + \frac{a_{N+1}}{2^N} + \cdots$$

$$a^* = a_1 + \frac{a_2}{2} + \frac{a_3}{2^2} + \cdots + \frac{a_N}{2^{N-1}} + \frac{b_1}{2^N} + \cdots$$

$$\therefore |a - a^*| = \left| \frac{a_{N+1} - b_1}{2^N} + \frac{a_{N+2} - b_2}{2^{N+1}} + \cdots \right| \Rightarrow$$

$$|a - a^*| \leq \frac{1}{2^N} + \frac{1}{2^{N+1}} + \frac{1}{2^{N+2}} + \cdots$$

$$\leq \frac{1}{2^N} \left[1 + \frac{1}{2} + \frac{1}{4} + \cdots \right]$$

$$\leq \frac{1}{2^N} \quad \frac{1}{1 - 1/2} \leq \frac{1}{2^{N-1}}$$

From (1.2.51) we have

$$\frac{dx}{d\alpha} = \frac{\pi}{2} \sin \pi\alpha \Rightarrow \left| \frac{dx}{d\alpha} \right| \leq \pi/2 \Rightarrow$$

$$|X^a - X^{a^*}| \leq (\pi/2) \, |a - a^*| \Rightarrow$$

$$\boxed{|X^a - X^{a^*}| \leq \frac{\pi}{2^N}}$$