$$X_{n+1} = 4\left\{(1/2)\left[1-\cos(2^n\phi_0)\right]\right\}\left\{(1/2)\left[1+\cos(2^n\phi_0)\right]\right\}$$

$$= 4\, X_n \left\{(1/2)\left[1+1-2X_n\right]\right\} \Rightarrow$$

$$\boxed{X_{n+1} = 4\, X_n\,(1-X_n)}$$

Much of the rest of this problem is self explanatory.

For example, $.01001001 00\cdots$

$$= \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \frac{1}{2^{11}} + \cdots$$

$$= \frac{1}{2^2}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \cdots\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \left(\frac{1}{8}\right)^3 + \cdots\right]$$

$$= \frac{1}{4}\,\frac{1}{1-1/8} = \frac{1}{4}\,\frac{1}{7/8} = \frac{1}{4}\,\frac{8}{7} = \frac{2}{7}$$

Suppose $\alpha_n = a_1 . a_2\, a_3\, a_4 \cdots$. Then

$2\alpha_n = a_1 a_2 . a_3 a_4 \cdots$, and

$\alpha_{n+1} = 2\,\alpha_n \bmod 2 = a_2 . a_3 a_4 \cdots$.