

DLN 1.2.5

1/1

We have $z_{n+1} = a + bz_n + cz_n^2$ with $c \neq 0$.

$$\text{Let } z = w/c - b/(2c) \Rightarrow$$

$$z_{n+1} = w_{n+1}/c - b/(2c) \Rightarrow$$

$$w_{n+1}/c - b/(2c) = a + b[w_n/c - b/(2c)] + c[w_n/c - b/(2c)]^2$$

$$\Rightarrow w_{n+1} = \frac{b}{2} + ac + b[w_n - b/2] + [w_n - b/2]^2$$

$$= \frac{b}{2} + ac + bw_n - \frac{b^2}{2} + w_n^2 - bw_n + \frac{b^2}{4}$$

$$= w_n^2 + \left(ac + \frac{b}{2} - \frac{b^2}{4} \right)$$

$$\Rightarrow w_{n+1} = w_n^2 - \left(\frac{b^2}{4} - \frac{b}{2} - ac \right)$$

$$\Rightarrow \boxed{w_{n+1} = w_n^2 - \mu} \quad \text{with}$$

$$\boxed{\mu = \frac{b^2}{4} - \frac{b}{2} - ac}$$