

So, we must check

$$x^2 + y^2 + x/2 \stackrel{?}{=} (1/2) \sqrt{x^2 + y^2}.$$

In polar coordinates this reads

$$\rho^2 + (1/2)\rho \cos \theta \stackrel{?}{=} \rho/2 \Rightarrow$$

$$\rho + (1/2) \cos \theta \stackrel{?}{=} (1/2) \Rightarrow$$

$$\rho \stackrel{?}{=} (1/2)(1 - \cos \theta), \text{ which we know}$$

is true. So the image of the circle

$|z| = 1$  is a cardioid. It is easy to

check that the interior of this circle maps into the interior of the cardioid.

Finally, if we write  $\gamma = 2 - e^{2\phi}$

for the circle on the right side of

Figure 1.2.5, we get

$$u'' = -(r-2)^2/4 = -(1/4)(1 - e^{2\phi})^2$$

$$= -(1/4)(1 - e^{2\phi})^2, \text{ which is the}$$

same equation as before. So, this