DLN 1.2.4

The complex logistics map is

$$\bar{z} = \gamma z (1-z).$$

Set $z = -(w/\gamma) + \frac{1}{2}$ and $\bar{z} = -\frac{\bar{w}}{\gamma} + \frac{1}{2} \Rightarrow$

$$-\frac{\bar{w}}{\gamma} + \frac{1}{2} = \gamma\left[-\frac{w}{\gamma} + \frac{1}{2}\right]\left[\frac{1}{2} + \frac{w}{\gamma}\right] \Rightarrow$$

$$\frac{\bar{w}}{\gamma} - \frac{1}{2} = \gamma\left[\left(\frac{w}{\gamma}\right)^2 - \frac{1}{4}\right] \Rightarrow$$

$$\bar{w} = \frac{\gamma}{2} + \gamma^2\left[\frac{w^2}{\gamma^2} - \frac{1}{4}\right] \Rightarrow$$

$$\bar{w} = w^2 + \frac{\gamma}{2} - \frac{\gamma^2}{4} \Rightarrow$$

$$\boxed{\bar{w} = w^2 - \mu \quad \text{with } \mu = \frac{\gamma^2}{4} - \frac{\gamma}{2}}$$

Evidently both $\pm w$ map to the same $\bar{w}$. Therefore the map is two-to-one.

In fact, given $\bar{w}$, let us solve for w. Doing so gives $w = \pm\sqrt{\bar{w} + \mu}$, which again shows that the map is two-to one.

Let us compute $\frac{d\bar{w}}{dw} = 2w$. By