

$$P(x) = x^3 - 2x^2 - \frac{(1+\lambda)}{\lambda}x + \frac{1}{\lambda^3} - \frac{1}{\lambda} = 0. \quad (7)$$

We know that $P(x)$ has $x_e \approx \frac{\lambda-1}{\lambda}$ as a root
(x_e is a fixed point of m , and therefore also of m^2).

Dividing out the factor $(x - x_e) = (x - \frac{\lambda-1}{\lambda})$ gives

$$\frac{P(x)}{(x - \frac{\lambda-1}{\lambda})} = x^2 - x \left(\frac{\lambda+1}{\lambda} \right) + \frac{\lambda+1}{\lambda^2} = Q(x). \quad (7)$$

The equation $Q(x) = 0$ has the solutions

$$x^{\pm} = \frac{\lambda+1}{2\lambda} \pm \frac{1}{2\lambda} \sqrt{(\lambda-3)(\lambda+1)}. \quad (8)$$

Note that $x^{\pm} \Big|_{\lambda=3} = \frac{3+1}{6} = \frac{4}{6} \approx \frac{2}{3} = \frac{\lambda-1}{\lambda} \Big|_{\lambda=3} = x_e \Big|_{\lambda=3}$.

Note also that $\frac{\partial x^{\pm}}{\partial \lambda} \Big|_{\lambda=3} = \pm \infty$. Thus, the

slope of the curves $x^{\pm}(\lambda)$ are ∞ at $\lambda=3$.

Evaluating at $\lambda = 3.449$ gives

$$x^+ - x^- = \frac{1}{\lambda} \sqrt{(\lambda-3)(\lambda+1)} \Big|_{\lambda=3.449} = .409 \dots$$