DLN1.2.2: Suppose we write $\bar{x} = \mathcal{M}x$, which for the logistics map is equivalent to the equation

$$\bar{x} = \lambda x(1-x). \qquad (1)$$

Let us also write $\bar{\bar{x}} = \mathcal{M}\bar{x}$, which is equivalent to the equation

$$\bar{\bar{x}} = \lambda \bar{x}(1-\bar{x}). \qquad (2)$$

If we write $\bar{\bar{x}} = \mathcal{M}\bar{x} = \mathcal{M}\mathcal{M}x = \mathcal{M}^2 x$, then substitution of (1) into (2) gives for $\mathcal{M}^2$ the relation

$$\bar{\bar{x}} = \lambda\,[\lambda x(1-x)]\,[1-\lambda x(1-x)]. \qquad (3)$$

Multiplying out gives the result

$$\bar{\bar{x}} = \lambda^2 x(1-x)\,[\lambda x^2 - \lambda x + 1]. \qquad (4)$$

For a fixed point of $\mathcal{M}^2$ we must have $\bar{\bar{x}} = x \Rightarrow$

$$x = \lambda^2 x(1-x)\,[\lambda x^2 - \lambda x + 1]. \qquad (5)$$

One solution of this is $x_e = 0$, which we already knew. Dividing both sides of (5) by x shows that the others must satisfy

$$1 = \lambda^2(1-x)\,[\lambda x^2 - \lambda x + 1]. \qquad (6)$$

Multiplying out and dividing by λ^3 gives the result