The logistic map is $\bar{x} = \lambda x(1-x)$. Let x_e be a fixed point, and consider a nearby point of the form

$$X = x_e + \delta \quad (1)$$

Where δ is small. Then we get

$$\bar{x} = m(x_e+\delta) = \lambda(x_e+\delta)(1-x_e-\delta)$$

$$= \lambda x_e(1-x_e) + \lambda\delta(1-2x_e) - \lambda\delta^2$$

$$= x_e + \lambda\delta(1-2x_e) - \lambda\delta^2$$

Ignore the δ^2 term, and examine the factor

$\lambda(1-2x_e)$ that multiplies δ.

Case i: $x_e = 0 \Rightarrow \lambda(1-2x_e) = \lambda \Rightarrow$

$$\bar{x} = x_e + \lambda\delta.$$

We see that successive points are closer to x_e and move away

if $\lambda < 1 \Rightarrow$ stability;

if $\lambda > 1 \Rightarrow$ instability.

Case ii: $x_e = \frac{\lambda-1}{\lambda} = 1 - \frac{1}{\lambda} \Rightarrow$

$$\lambda(1-2x_e) = \lambda\left(1-2+\frac{2}{\lambda}\right) = (2-\lambda)$$

We see that $(2-\lambda) > 1$ if $\lambda < 1 \Rightarrow$ instability;

$|(2-\lambda)| < 1$ if $\lambda \in [1,3] \Rightarrow$ stability