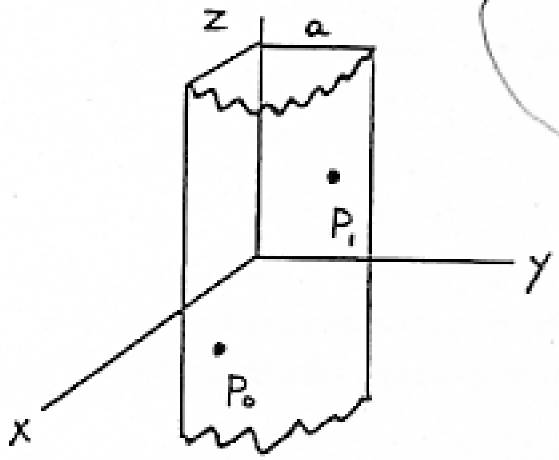
8. (20 pts) Consider the cylinder of radius a shown below. Use cylindrical coordinates ρ, ϕ, and z. Assume the two points P_0 and P_1 on the cylinder have coordinates z_0, ϕ_0 and z_1, ϕ_1 respectively. Find the shortest geodesic on the cylinder joining P_0 and P_1. (Assume $|\phi_1 - \phi_0| < \pi$)

$$\boxed{z = \frac{z_0 \phi_1 - z_1 \phi_0}{\phi_1 - \phi_0} + \frac{z_1 - z_0}{\phi_1 - \phi_0}\,\phi}$$

or



$$\boxed{z(\phi) = z_0 + \left(\frac{z_1 - z_0}{\phi_1 - \phi_0}\right)(\phi - \phi_0)}$$

A Helix

$ds^2 = d\rho^2 + dz^2 + \rho^2\, d\phi^2$ in cylindrical coordinates,

$\rho = a \Rightarrow ds^2 = dz^2 + a^2\, d\phi^2$ on surface of cylinder

$\Rightarrow ds = \sqrt{(z')^2 + a^2}\; d\phi$ with $z' = dz/d\phi$

$\Rightarrow \mathcal{L} = \sqrt{(z')^2 + a^2}$ + $\frac{\partial L}{\partial z} = 0 \Rightarrow \frac{d}{d\phi}\frac{\partial \mathcal{L}}{\partial z'} = 0$

$\Rightarrow$ cnst $= \frac{\partial \mathcal{L}}{\partial z'} = \frac{z'}{\sqrt{z'^2 + a^2}} \Rightarrow z' =$ cnst $\Rightarrow \boxed{z = \alpha + \beta\phi}$

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Use $z(\phi_0) = z_0$ + $z(\phi_1) = z_1$ to determine α and β.