

3. (4 pts) Consider the motion of a particle in an attractive central force with $F(r) \sim r^n$. All bound orbits are closed when

$$n = 1 \text{ or } -2 \quad \left[\text{Harmonic oscillator or Kepler problem.} \right]$$

Circular orbits are stable provided

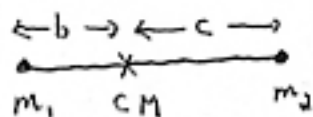
$$n > -3$$

In the Kepler problem, for a fixed angular momentum the orbit having the least energy is

_____ elliptic _____ parabolic
 _____ X _____ circular _____ hyperbolic

4. (10 pts) Consider a double star system consisting of two stars having masses m_1 and m_2 . They both move about their common center of mass (which is at rest) in *circular* orbits. Their mutual separation has a constant value a , and the gravitational constant is G . Their period is given by

$$\tau = 2\pi \left[\frac{a^3}{(m_1 + m_2) G} \right]^{1/2}$$



(1) $m_1 b = m_2 c$, (2) $b + c = a$, Newton says

$$\underbrace{m_2 \omega^2 c}_{\text{Acceleration of } m_2} = \underbrace{\frac{m_1 m_2 G}{a^2}}_{\text{Force on } m_2}$$

$$(1) + (2) \Rightarrow c \left(1 + \frac{m_2}{m_1} \right) = a$$

$$\Rightarrow m_2 c = \frac{m_1 m_2 a}{m_1 + m_2} \Rightarrow \omega^2 = \frac{(m_1 + m_2) G}{a^3} \Rightarrow \omega = \left[\frac{(m_1 + m_2) G}{a^3} \right]^{1/2}$$

Finally, $\tau = 2\pi/\omega$.