$$m\ddot{\vec{r}} = q(\vec{E} + \vec{v}\times\vec{B}), \quad q = -e \text{ with } e>0 \Rightarrow$$

$$m\ddot{\vec{r}} = e(E\hat{e}_y + \vec{v}\times B\hat{e}_z) \text{ or } \ddot{\vec{r}} = \frac{eE}{m}\hat{e}_y + \frac{eB}{m}\vec{v}\times\hat{e}_z \Rightarrow$$

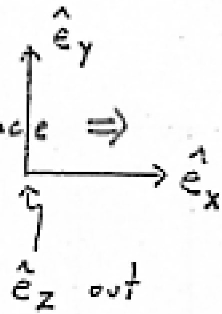
$$(i)\ \ddot{x} = \frac{eB}{m}\dot{y}, \quad (ii)\ \ddot{y} = \frac{eE}{m} - \frac{eB}{m}\dot{x}, \quad (iii)\ \ddot{z} = 0$$

9. (30 pts) An electron is emitted at $t = 0$ with negligible velocity from a hot filament into the area between the plates of a parallel plate condenser as shown below:

Solve (iii) first: $\ddot{z} = 0 \Rightarrow z = z^0 + \dot{z}^0 t \Rightarrow$, using the initial conditions, $z = 0$. Next integrate (i) once $\Rightarrow$

$\dot{x} = \frac{eB}{m} y + \text{cnst}$ + using initial conditions $\Rightarrow$ cnst $= 0$

$\Rightarrow \dot{x} = \frac{eB}{m} y$ (iv)



Substitute (iv) into (ii) $\Rightarrow \ddot{y} = \frac{eE}{m} - \omega^2 y$ with $\omega = \frac{eB}{m}$

Locate the origin at the filament. A potential difference is maintained between the plates to produce a uniform electric field in the negative "y" direction,

$$E = -Ee_y.$$

$y = \frac{eE}{m\omega^2} + a\cos\omega t + b\sin\omega t$

In addition, a uniform magnetic field is applied in the negative "z" direction,

$$B = -Be_z.$$

+ initial conditions $\Rightarrow a = -eE/m\omega^2$, $b = 0$

Find the subsequent motion, assuming the electron does not hit the opposite condenser plate.

$x(t) = \frac{eE}{m\omega^2}(\omega t - \sin\omega t) = \frac{E}{B}t - \frac{Em}{eB^2}\sin\frac{eBt}{m}$

$y(t) = \frac{eE}{m\omega^2}(1 - \cos\omega t) = \frac{Em}{eB^2}\left(1 - \cos\frac{eBt}{m}\right)$

$z(t) = 0$

$\omega = \frac{eB}{m}$, $e>0$

MORE ROOM ON NEXT PAGE

$y = \frac{eE}{m\omega^2}(1 - \cos\omega t) \Rightarrow$ 10 $\dot{x} = \frac{eE}{m\omega}(1 - \cos\omega t) \Rightarrow$

put into (iv)

$x = \frac{eE}{m\omega}t - \frac{eE}{m\omega^2}\sin\omega t + \text{cnst}$

+ cnst = 0 using the initial conditions,