Equilibrium at $y=0$. Expand V about $y=0 \Rightarrow$

$$V(y) = -\frac{2mMG}{d}\left\{1 - \frac{y^2}{2d^2} + \text{hot}\right\} \Rightarrow F_y = -\frac{\partial V}{\partial y}$$

$$= -\frac{2mMG}{d}\frac{y}{d^2} + \text{hot}. \quad m\ddot{y} = F_y \Rightarrow$$

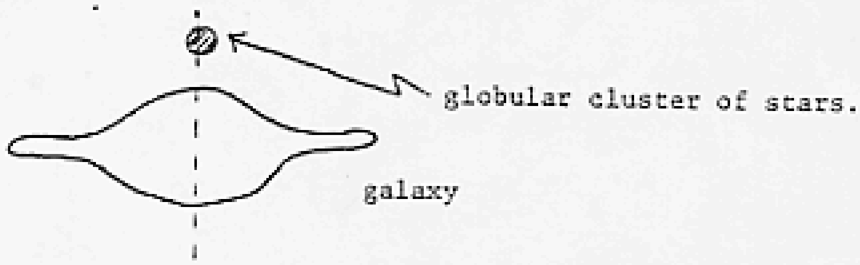
$$m\ddot{y} + \frac{2mMG}{d^3}y = 0 + \text{hot} \Rightarrow$$

$$\ddot{y} + \frac{2MG}{d^3}y = 0$$

(b) (5 pts) Find the equilibrium position of the bead, and calculate the frequency of small oscillations about this position.

$$\omega = \sqrt{\frac{2MG}{d^3}}$$

(c) (5 pts) Our galaxy, when viewed side-on, looks roughly as shown below:



It has been observed that there are concentrated globs of stars, called globular clusters, which are above and below the galactic plane. The clusters may contain as many as 100,000 stars. These clusters appear from Doppler-shift measurements to be "bouncing" back and forth across the galactic plane much as the bead moves up and down on the wire in parts a and b. Taking $2M \sim 10^{44}$ grams (the mass of our galaxy), and $d \sim 10^{23}$ cm, (the radius of our galaxy), estimate the time τ in seconds required for a cluster to go through one oscillation using the frequency found in part b. The gravitational constant is $G \sim 10^{-7}$ cm^3/(gram sec^2).

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{10^{69}}{10^{44}\times 10^{-7}}} \simeq 2\pi\sqrt{10^{32}}$$

$$\Rightarrow \tau = 2\pi \times 10^{16}\ \text{sec} \simeq 1/6 \text{ age of Universe}$$

(More Room for Work on Next Page)