PART II

9. (15 pts) Consider the differential equation $\dot{y} = y^2 + t$ with the initial condition $y(0) = 1$. This equation can be written in the form $\dot{y} = f(y,t)$ with $f(y,t) = y^2 + t$.

(a) (3 pts) Do you expect a solution to exist and be unique in the vicinity of $t = 0$ and $y = 1$? If so, why? Yes:

$\frac{\partial f}{\partial t} = 1$; $\frac{\partial f}{\partial y} = 2y$ and $\frac{\partial^2 f}{\partial y^2} = 2 \Rightarrow$

Both $\frac{\partial f}{\partial t}$ and $\frac{\partial^2 f}{\partial y^2}$ are continuous near $t=0$ and $y=1$ (actually everywhere), and the criteria for the existence and uniqueness theorem are met.

(b) (12 pts) Using Euler's crude method with a step size of $h = .1$, fill out the table below to find y at the time $t = .2$.

n	t^n	y^n	f^n
0	0	1	1
1	.1	1.1	1.31
2	.2	1.231	not needed

Euler says $y^{n+1} = y^n + hf^n$ where $f^n = f(y^n, t^n)$.

$y^0 = 1$ and $t^0 = 0 \Rightarrow f^0 = 1^2 + 0 = 1$; $y^1 = y^0 + hf^0 = 1 + h = 1.1$;

$f^1 = (y^1)^2 + t^1 = (1.1)^2 + .1 = 1.31$; $y^2 = y^1 + hf^1$

$= (1.1) + (.1)(1.31) = 1.1 + .131 = 1.231$