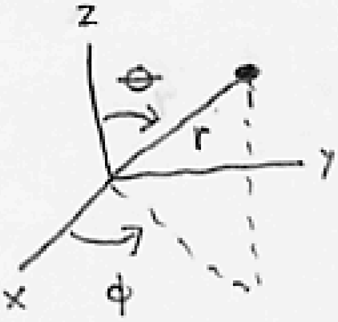
7. (6 pts) A particle of mass m moves in a potential field $V(r,\theta,\phi;t)$. Its Lagrangian in spherical coordinates is

$$\mathcal{L}(r\ \theta\ \phi;\ \dot{r}\ \dot{\theta}\ \dot{\phi};\ t) = \frac{m}{2}\left[(\dot{r})^2 + r^2(\dot{\theta})^2 + r^2\sin^2\theta(\dot{\phi})^2\right] - V$$

and the canonically conjugate momenta are explicitly given by



$$p_r = \frac{\partial \mathcal{L}}{\partial \dot{r}} = m\dot{r}$$

$$p_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m r^2 \dot{\theta}$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m r^2 \sin^2\theta\ \dot{\phi}$$

8. (6 pts) For a single particle the Virial Theorem reads $V = \frac{1}{2}kx^2$, $F = -kx$

$$\langle 2T\rangle = -\langle \boldsymbol{r}\cdot\boldsymbol{F}\rangle. \quad \Rightarrow \langle 2T\rangle = \langle x\,kx\rangle = \langle 2V\rangle$$

Apply this theorem to a one-dimensional harmonic oscillator described by a potential V and having total energy E:

$T + V = E \Rightarrow$

$\langle T\rangle + \langle V\rangle = E$

(a) Relate $\langle T\rangle$ and $\langle V\rangle$.

$$\langle T\rangle = \langle V\rangle$$

(b) Give a formula for $\langle T\rangle$ in terms of E.

$$\langle T\rangle = E/2$$

(c) Give a formula for $\langle V\rangle$ in terms of E.

$$\langle V\rangle = E/2$$