Physics 601
Dr. Dragt

First Hourly Exam
7 October 2002

Solution

PART I

1. (5 pts) Let $t^n = t^0 + nh$, $y^n = y(t^n)$, and define a backwards difference operator ∇ by $\nabla y^n = y^n - y^{n-1}$. Let $D = d/dt$. Then D is related to ∇ by

$D = -h^{-1} \log(1 - \nabla)$

2. (7 pts) Consider the differential equation

$$\dot{y} = f(y, t)$$

with $f^n = f(y^n, t^n)$, and the integration formulas

$$y^{n+1} = y^n + h \sum_0^N a_k \nabla^k f^{n+1}, \qquad \text{A}$$

$$y^{n+1} = y^n + h \sum_0^N b_k \nabla^k f^n. \qquad \text{B}$$

Formula ______B______ is a predictor formula
and formula ______A______ is a corrector formula.

I expect the integration error in making a single step to be of order h^m with

$m = N + 2$

(Suppose $N = 0 \Rightarrow$ $\vec{y}^{n+1} = \vec{y}^n + h b_0 f^n$, which obviously has an error of order h^2.)

3. (3 pts) Expand out:

$\nabla^3 y^n = \vec{y}^n - 3\vec{y}^{n-1} + 3\vec{y}^{n-2} - \vec{y}^{n-3}$