See Section 1.3 (The motion of a single particle) in Chapter 1 (ELEMENTARY CONCEPTS) of Dragt notes on CLASSICAL MECHANICS

12. (15pts) Name and state below the three Conservation Laws in Newtonian Mechanics for Single Particle Motion and describe under what conditions they hold. Make whatever definitions are required.

(i) (4 pts) Linear Momentum: $\vec{p} = m\vec{v}$. Newton $\Rightarrow$ $m\vec{a} = \vec{F} \Rightarrow \dot{\vec{p}} = \vec{F}$ and $\vec{F} = 0 \Rightarrow$

$\vec{p}$ = constant vector $\Leftrightarrow$ Linear momentum conservation

(ii) (5 pts) Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$ and torque $\vec{N} = \vec{r} \times \vec{F}$. Newton $\Rightarrow \dot{\vec{p}} = \vec{F} \Rightarrow$ $\dot{\vec{L}} = \vec{N}$ and $\vec{N} = 0 \Rightarrow$

$\vec{L}$ = constant vector $\Leftrightarrow$ Angular Momentum Conservation

(iii) (6 pts) Energy: Assume $\vec{F}(\vec{r}, \vec{v}, t) = \vec{F}_1(\vec{r}) + \vec{F}_2(\vec{r}, \vec{v}, t)$ with $\vec{\nabla} \times \vec{F}_1 = 0$ and $\vec{v} \cdot \vec{F}_2 = 0$. Define $T = \frac{1}{2} m \vec{v} \cdot \vec{v}$ and $V(\vec{r})$ such that $\vec{F}_1 = -\vec{\nabla} V$. Then $\dot{\vec{p}} = \vec{F} \Rightarrow$

$E = T + V$ = constant $\Leftrightarrow$ Energy Conservation