PHYS420 (Spring 2006) Homework # 2 Solutions

Problem 1.5

This is a case of dilation. $T = \gamma T'$ in this problem with the proper time $T' = T_0$

$$T = [1 - (\frac{v}{c})^2]^{-1/2} T_0 \Rightarrow \frac{v}{c} = [1 - (\frac{T_0}{T})^2]^{1/2} \tag{1}$$

in this case $T = 2T_0$, $v = [1 - [\frac{T_0/2}{T_0}]^2]^{1/2} = [1 - (\frac{1}{4})]^{1/2}$ therefore $v = 0.866c$.

Problem 1.6

This is a case of length contraction. $L = \frac{L'}{\gamma}$ in this problem the proper length $L' = L_0$,

$$L = [1 - (\frac{v}{c})^2]^{-1/2} L_0 \Rightarrow \frac{v}{c} = [1 - (\frac{L_0}{L})^2]^{1/2} \tag{2}$$

in this case $L = \frac{L_0}{2}$, $v = [1 - (\frac{L_0}{L})^2]^{1/2} c = [1 - \frac{1}{4}]^{1/2}$, therefore $v = 0.866c$.

Problem 1.14

(a) Only the x component of L_0 contracts.

$$L_{x'} = L_0 \cos\theta_0 \Rightarrow L_x = \frac{L_0 \cos\theta_0}{\gamma} \tag{3}$$

$$L_{y'} = L_0 \sin\theta_0 \Rightarrow L_y = L_0 \sin\theta_0 \tag{4}$$

$$L = [L_x^2 + L_y^2]^{1/2} = [(\frac{L_0 \cos\theta_0}{\gamma})^2 + (L_0 \sin\theta_0)^2]^{1/2} \tag{5}$$

$$= L_0[\cos{\theta_0}^2 (1 - \frac{v^2}{c^2}) + \sin{\theta_0}^2]^{1/2} = L_0[1 - \frac{v^2}{c^2} \cos{\theta_0}^2]^{1/2} \tag{6}$$

(b) As seen by the stationary observer, $\tan\theta = \frac{L_y}{L_x} = \frac{L_0 \sin\theta_0}{L_0 \cos\theta_0/\gamma} = \gamma \tan\theta_0$.

Problem 1.18

(a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{source} \frac{c+v}{c}$ and, if f is the frequency of the reflected wave, $f = f_c \frac{c}{c-v}$. Combining these equations gives $f = f_{source} \frac{c+v}{c-v}$.

(b) Using the above result, $f(c - v) = f_{source}(c + v)$, which gives

$$(f - f_{source})c = (f + f_{source})v \approx 2 f_{source} v. \tag{7}$$

The beat frequency is then $f_{beat} = f - f_{source} = \frac{2 f_{source} v}{c} = \frac{2v}{\lambda}$.

(c)

$$f_{beat} = \frac{2(30.0m/s)(10.0 \times 10^9 Hz)}{3.00 \times 10^8 m/s} = \frac{2(30.0m/s)}{0.030m} = 2000Hz = 2.00kHz \quad (8)$$

$$\lambda = \frac{c}{f_{source}} = \frac{3.00 \times 10^8 m/s}{10.0 \times 10^9 Hz} = 3.00cm \quad (9)$$

(d)

$$v = \frac{f_{beat}\lambda}{2} \quad (10)$$

$$\Delta v = \frac{\Delta f_{beat}\lambda}{2} = \frac{(5Hz)(0.030m)}{2} = 0.0750m/s \approx 0.2mi/h \quad (11)$$

Problem 1.24

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0 - 0.90c}{1 - (0)(0.90c)/c^2} = -0.90c \quad (12)$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)} = \frac{0 - 0.90c}{[1 - 0.81]^{-1/2}} \approx -0.392c \quad (13)$$

The speed of A as measured by B is

$$u_{AB} = [(u'_x)^2 + (u'_y)^2]^{1/2} = [(-0.90c)^2 + (-0.392c)^2]^{1/2} = 0.982c \quad (14)$$

Classically, $u_{AB} = 1.3c$.

Problem 1.36

Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0, t_1 = 0, x_2 = 0, t_2 = 3\mu s$. Let Mark be fixed in reference frame S' and give the events coordinate $x'_1 = 0, t'_1 = 0, t'_2 = 9\mu s$.

(a) Then we have

$$t'_2 = \gamma(t_2 - \frac{v}{c^2}x_2) = 9\mu s = \frac{1}{\sqrt{1 - v^2/c^2}}(3\mu s - 0) \Rightarrow \sqrt{1 - v^2/c^2} = 1/3 \Rightarrow v = 0.943c. \quad (15)$$

(b)

$$x'_2 = \gamma(x_2 - vt_2) = 3(0 - 0.943c \times 3 \times 10^{-6}s) = 2.55 \times 10^3 m. \quad (16)$$

Problem 1.40

The slope of the ct'- axis must be such that the origin O' moves at velocity +v in frame S. Using Equation 1.31, we find the slope of the ct' axis must be $\frac{c}{+v}$ or between $+\infty$ and +1. Because a light pulse moving in the +x direction starting from the origin is described by the equation x' = ct' in frame S', the light pulse line must bisect the angle between the ct' - axis and the x'- axis. Therefore the x'-axis is rotated counterclockwise from the x-axis the same amount as the ct'-axis is rotated clockwise from the ct-axis.