PHYS 420 – SPRING 2006

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PROBLEM SET # 5 - SOLUTION

3-3 (a) The total energy of a simple harmonic oscillator having an amplitude A is $\frac{kA^2}{2}$, therefore, $E = \frac{kA^2}{2} = (25\ \text{N/m})\frac{(0.4\ \text{m})^2}{2} = 2.0\ \text{J}$. The frequency of oscillation will be $f = \left(\frac{1}{2\pi}\right)\left(\frac{k}{m}\right)^{1/2} = \left(\frac{1}{2\pi}\right)\left(\frac{25}{2}\right)^{1/2} = 0.56\ \text{Hz}$.

(b) If energy is quantized, it will be given by $E_n = nhf$ and from the result of (a) there follows $E_n = nhf = n\left(6.63\times10^{-34}\ \text{J s}\right)(0.56\ \text{Hz}) = 2.0\ \text{J}$. Upon solving for n one obtains $n = 5.4\times10^{33}$. Of course we cannot have 5.4 x10^{33} photons we either have 5 x10^{33} photons or 6 x10^{33} photons.

(c) The energy carried away by one quantum of charge in energy will be $E = hf = \left(6.63\times10^{-34}\ \text{J s}\right)(0.56\ \text{Hz}) = 3.7\times10^{-34}\ \text{J}$.

3-4 (a) From Stefan's law, one has $\frac{P}{A} = \sigma T^4$. Therefore,

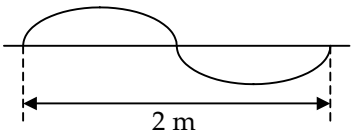
$$\frac{P}{A} = \left(5.7\times10^{-8}\ \text{W/m}^2K^4\right)(3000\ \text{K})^4 = 4.62\times10^{6}\ \text{W/m}^2.$$

(b) $$A = \frac{P}{4.62\times10^{6}\ \text{W/m}^2} = \frac{75\ \text{W}}{4.62\times10^{6}\ \text{W/m}^2} = 16.2\ \text{mm}^2.$$

3-7 (a) In general, $L = \frac{n\lambda}{2}$ where $n = 1,\ 2,\ 3,\ \ldots$ defines a mode or standing wave pattern with a given wavelength. As we wish to find the number of possible values of n between 2.0 and 2.1 cm, we use $n = \frac{2L}{\lambda}$

$$n(2.0\ \text{cm}) = (2)\frac{200}{2.0} = 200$$
$$n(2.1\ \text{cm}) = (2)\frac{200}{2.1} = 190$$
$$|\Delta n| = 10$$

As n changes by one for each allowed standing wave, there are 10 standing waves of different wavelength between 2.0 and 2.1 cm.



(b) The number of modes per unit wavelength per unit length is $\frac{\Delta n}{L\Delta\lambda} = \frac{10}{0.1}(200) = 0.5 \text{ cm}^{-2}$.

(c) For short wavelengths n is almost a continuous function of λ. Thus one may use calculus to approximate $\frac{\Delta n}{L\Delta\gamma} = \left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right)$. As $n = \frac{2L}{\lambda}$, $\left|\frac{dn}{d\lambda}\right| = \frac{2L}{\lambda^2}$ and $\left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right) = \frac{2}{\lambda^2}$. This gives approximately the same result as that found in (b): $\left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right) = \frac{2}{\lambda^2} = \frac{2}{(2.0 \text{ cm})^2} = 0.5 \text{ cm}^{-2}$.

(d) For short wavelengths n is almost a continuous function of λ, $n = \frac{2L}{\lambda}$ is a discrete function.

3-20 $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - K_{\max}$;

First Source: $\phi = \frac{hc}{\lambda} - 1.00 \text{ eV}$.

Second Source: $\phi = \frac{hc}{\frac{\lambda}{2}} - 4.00 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV}$.

As the work function is the same for both sources (a property of the metal), $\frac{hc}{\lambda} - 100 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV} \Rightarrow \frac{hc}{\lambda} = 3.00 \text{ eV}$ and $\phi = \frac{hc}{\lambda} - 1.00 \text{ eV} = 3.00 \text{ eV} - 1.00 \text{ eV} = 2.00 \text{ eV}$.

The first x-ray intensity maximum in the diffraction pattern occurs at $\theta = 6.41°$. To determine d use the Bragg diffraction condition $n\lambda = 2d\sin\theta$ for $n = 1$.

$$d = \frac{\lambda}{2\sin\theta} = \frac{0.626 \text{ Å}}{2\sin 6.41°} = 2.80 \text{ Å} = 2.80 \times 10^{-8} \text{ cm}.$$

From Figure P3.39 there are $(4)\left(\frac{1}{8}\right)\text{Cl}^-$ and $(4)\left(\frac{1}{8}\right)\text{Na}^+$ ions per primitive cell. This works out to half a NaCl formula unit per primitive cell. The formula weight of NaCl is 58.4 g/mole. Setting the mass per unit volume of the primitive cell equal to the density we have $\frac{(58.4 \text{ g/mole})(\text{formula unit})}{2d^3 N_A} = \rho$ where N_A is the number of formula units per mole (Avogadro's number). So

$$N_A = \frac{(58.4\ \text{g/mol})(\text{formula unit})}{2d^3}$$

$$\rho = \frac{(58.4\ \text{g/mol})(\text{formula unit})}{2\left(2.8\times10^{-8}\ \text{cm}\right)^3\left(2.17\ \text{g/cm}^3\right)}$$

$$N_A = 6.13\times10^{23}\ \text{formula units/mole}$$

3-44 Each emitted electron requires an energy

$$hf = \frac{1}{2}mv^2 + \phi = \left(\frac{9.11\times10^{-31}\ \text{kg}}{2}\right)\left(4.2\times10^5\ \text{m/s}\right)^2 + (3.44\ \text{eV})\left(1.6\times10^{-19}\ \text{J/eV}\right)$$

$\Delta E = 6.3\times10^{-19}$ J per emitted electron.

Therefore, with an incident intensity of $\frac{0.055\ \text{J/m}^2}{\text{s}} = \frac{5.5\times10^{-6}\ \text{J/cm}^2}{\text{s}}$, the number of electrons emitted per cm^2 per second is

$$\text{electron flux} = \frac{\frac{5.5\times10^{-8}\ \text{J/cm}^2}{\text{s}}}{6.3\times10^{-19}\ \text{J/emitted electron}} = \frac{8.73\times10^{12}}{\text{cm}^2/\text{s}}.$$