Phys 404
Spring 2010
Homework 5, CHAPTER 5
Due Thursday, March 25, 2010 @ 12:30 PM

General hint: Problems 1 – 3 involve classical statistical mechanics. In this case, the sums over quantum states are replaced by integrals over all position and momentum components, so that, for example,

$$\langle X \rangle = \frac{\frac{1}{h^{Nd}} \int \ldots \int X(\vec{p},\vec{q}) e^{\left(\frac{-H(\vec{p},\vec{q})}{\tau}\right)} (d\vec{p})^N (d\vec{q})^N}{Z} \qquad Z = \frac{1}{h^{Nd}} \int \ldots \int e^{\left(\frac{-H(\vec{p},\vec{q})}{\tau}\right)} (d\vec{p})^N (d\vec{q})^N$$

Here N is the number of particles, d is the spatial dimensionality of the problem, H(**p**, **q**) is the Hamiltonian (total energy) in terms of the vector coordinates (**q**) and momenta (**p**) of all the particles, and h is Planck's constant. Note that there is one integral for each component of momentum and position for each particle, so that there are 2Nd integral signs indicated by the $\int \ldots \int$ in the formulas. See the Lecture 13 summary for a review of classical statistical mechanics.

1. A pendulum hangs under gravity, has length *L* and mass *m*, and makes an angle θ with the vertical direction. Assuming that the amplitude of oscillation is small, find $<\theta>$, $<\theta^2>$, when the temperature of the pendulum is τ. Hint: start with the Hamiltonian (total energy in terms of momentum and coordinate), and expand for small angle.

2. Referring to problem 1, what value of *mL* will give $(<\theta^2>)^{1/2} = 0.001$ radian when the pendulum is at T = 300 K?

3. Consider a classical *N*-particle system with Hamiltonian given by

$$H = \sum_{i=1}^{N} (p_{xi}^2 + p_{yi}^2 + p_{zi}^2)/2m_i \quad + \quad V(\mathbf{r_1}, \mathbf{r_2}, \ldots \mathbf{r_N})$$

Show that the partition function *Z* can be separated into kinetic and potential energy parts, $Z = Z_{kin}Z_{pot}$, and that the Z_{kin} term is the partition function for an ideal gas. This result is useful for studying the statistical mechanics of liquids, because Z_{pot} depends only on the positions of the particles.

4. K+K, Chapter 5, Problem 1 Hint: Assume that the gas rotates with the centrifuge, as it will if there is any interaction coupling the gas molecules to each other and the walls. If these interactions are weak, you may use the internal chemical potential for an ideal gas. In the rotating frame, there is a fictitious force, which leads to an effective potential energy which depends on r and ω.

5. K+K, Chapter 5, Problem 2

6. K+K, Chapter 5, Problem 4 Hint: Use the factor of 10^4 in the second line of the problem as a given.