

a) In a centrifuge rotating with angular frequency ω , a molecule of mass m_1 feels an effective potential energy $-m_1 r^2 \omega^2 / 2$ when it is a distance r away from the center of the centrifuge. The molecule is part of an ideal gas with internal chemical potential $\mu_{\text{int}} = \tau \log(n_1/n_{Q1})$, where n_1 is the number of molecules per unit volume and $n_{Q1} = (m_1 \tau / 2\pi \hbar^2)^{3/2}$. Show that

$$n_1(r) = n_1(0) \exp\left(\frac{m_1 r^2 \omega^2}{2\tau}\right)$$

b) When a gas contains two different isotopes 1 and 2, the formula from part a) applies to each isotope separately with appropriate choice of subscripts. Derive a formula for the enrichment factor $[n_1(R)/n_2(R)] / [n_1(0)/n_2(0)]$, which tells how much the concentration ratio is changed between the center of the centrifuge (0) and the outer radius of the centrifuge (R).

c) If gas from radius R of one centrifuge is fed into the center of a second centrifuge, and the process is repeated many times, how many centrifuges N are required to make the final concentration ratio $n_{1f}/n_{2f} = 0.5$? Assume that the gas 1 is the heavier gas, and the original concentration ratio is $n_{1i}/n_{2i} = 0.01$. Do not bother to plug numbers into this part, just derive the formula.