PHY 402 - FALL 2014
HW 10 - SOLUTION

I. i) "-AT rest at the origin" is a funny way to say that we don't care about the orbital degrees of freedom ($\hat{x}$, $\hat{y}$ and $\hat{z}$). Thus the hamiltonian of the system is

$$\hat{H} = \frac{eg}{2m}\,\mathbf{B}\cdot\hat{\mathbf{S}} = \frac{eg}{2m}B_0\left[\sin\alpha\cos\omega t\,\hat{S}_x + \sin\alpha\sin\omega t\,\hat{S}_y + \cos\alpha\,\hat{S}_z\right]$$

$g \simeq 2$

The matrix of $\hat{H}$ in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis is:

$$H = \frac{egB_0\hbar}{2m2}\begin{pmatrix} \cos\alpha & \sin\alpha\, e^{-i\omega t} \\ \sin\alpha\, e^{i\omega t} & -\cos\alpha \end{pmatrix}$$

$$i\hbar\frac{d}{dt}|t\rangle = \hat{H}|t\rangle \Rightarrow i\hbar\frac{d}{dt}\begin{pmatrix} \chi_\uparrow(t) \\ \chi_\downarrow(t) \end{pmatrix} = \frac{egB_0\hbar}{4m}\begin{pmatrix} \cos\alpha & \sin\alpha\, e^{-i\omega t} \\ \sin\alpha\, e^{i\omega t} & -\cos\alpha \end{pmatrix}\begin{pmatrix} \chi_\uparrow(t) \\ \chi_\downarrow(t) \end{pmatrix}$$

Verifying that the proposed solution indeed solves the equation above is a matter of derivatives and matrix multiplication better left to computers (see the mathematica file attached)

ii) ~~We need the matrix element~~ We split $\hat{H}$ as:

$$\hat{H} = \underbrace{\frac{egB_0}{2m}\cos\alpha\,\hat{S}_z}_{\hat{H}_0} + \underbrace{\frac{egB_0}{2m}\sin\alpha\left[\cos\omega t\,\hat{S}_x + \sin\omega t\,\hat{S}_y\right]}_{\hat{V}}$$

eigenstates of $\hat{H}_0$: $|\uparrow\rangle$ w/ eigenvalue $\frac{egB_0\hbar}{2m}$
$|\downarrow\rangle$ w/ eigenvalue $-\frac{eB_0\hbar}{2m}$

We need the matrix element

$$\langle\downarrow|\hat{V}|\uparrow\rangle = \langle\downarrow|\frac{egB_0}{2m}\sin\alpha\left[\cos\omega t\,\hat{S}_x + \sin\omega t\,\hat{S}_y\right]|\uparrow\rangle$$

$$= \frac{egB_0\hbar\sin\alpha}{2m\,2}\underbrace{(\cos\omega t + i\sin\omega t)}_{e^{i\omega t}}$$

$$P = \frac{1}{\hbar^2}\left|\int_0^t dt'\,\frac{eB_0\hbar}{2m}\sin\alpha\,\underbrace{e^{i\omega t + \frac{eB_0}{m}t}}_{e^{i(\omega+\omega_1)t}}\right|^2$$

$$= \underbrace{\frac{e^2B_0^2}{4m^2}}_{\omega_1^2}\,4\,\frac{\sin^2(\omega+\omega_1)t/2}{(\omega+\omega_1)^2}\sin^2\alpha$$

Expanding the solution in i) in powers of $\sin\alpha$ (up to order $\sin^2\alpha$) we get the same answer (see the mathematica file attached.