QUANTUM PHYSICS I

PROBLEM SET 3

due September 19, before class

A. Pauli Matrices, $\epsilon-$tensor, ...

Show that

$$
\begin{aligned}
&i)\ [\sigma_i, \sigma_j] = i2\sum_k \epsilon_{ijk}\sigma_k \\
&ii)\ \{\sigma_i, \sigma_j\} = 2\delta_{ij} \\
&iii)\ \sigma_i\sigma_j = \delta_{ij} + i\sum_k \epsilon_{ijk}\sigma_k \\
&iv)\ \mathrm{tr}(\sigma_i) = 0 \\
&v)\ \mathrm{tr}(\sigma_i\sigma_j) = 2\delta_{ij} \\
&vi)\ \mathbf{v}.\sigma\ \mathbf{w}.\sigma = \mathbf{v}.\mathbf{w} + i(\mathbf{v}\times\mathbf{w}).\sigma \\
&vii)\ \mathbf{v}.\mathbf{w} = \sum_i v_i w_i \\
&viii)\ (\mathbf{v}\times\mathbf{w})_k = \sum_{ij} \epsilon_{ijk} v_i w_j \\
&ix)\ \sum_k \epsilon_{ijk}\epsilon_{i'j'k} = \delta_{ii'}\delta_{jj'} - \delta_{ij'}\delta_{ji'} \\
&x)\ \mathbf{v}\times(\mathbf{w}\times\mathbf{u}) = \mathbf{v}.\mathbf{u}\ \mathbf{w} - \mathbf{v}.\mathbf{u}\ \mathbf{w} \\
&xi)\ \mathbf{v}.(\mathbf{w}\times\mathbf{u}) = \mathbf{u}.(\mathbf{v}\times\mathbf{w})
\end{aligned}
\tag{1}
$$

where σ_i are the three Pauli matrices, the indices i, j, k go from 1 to 3, $\mathbf{v}, \mathbf{w}, \mathbf{u}$ are there-dimensional vectors. By taking one of the vectors to be the ∇ operator, all vector calculus identities can be proven by this method (you may enjoy proving some of them so you won't ever have to look them up again).

B. Even more bra-ketology

i) Let $\hat{A} = |\psi\rangle\langle\psi|$, for some $|\psi\rangle$ such that $\langle\psi|\psi\rangle = 1$. Compute

$$\cos(\lambda\hat{A}) = \sum_{n=0, n=even}^{\infty} \frac{(\lambda\hat{A})^n}{n!} =? \tag{2}$$

ii) Argue that any hermitian operator $\hat{A}$ can be written as

$$\hat{A} = \sum_n a_n |n\rangle\langle n|, \tag{3}$$

where $|n\rangle$ are its eigenvectors, a_n the corresponding eigenvalues and the sum is over all eigenvectors.

C. Spin

A spin-1/2 particle is initially in the state

$$|\psi\rangle = \frac{|+\rangle + i|-\rangle}{\sqrt{2}}, \tag{4}$$

where $|\pm\rangle$ are the spin up and down states (along the z-axis). By means of magnetic fields the spin of the particle is rotated around the z-axis by an angle of π. At this point the x-component of the spin is measured. What are the possible outcomes and with which probabilities?