Phys 402
Spring 2009
Homework 10
Due Friday, May 8, 2009 @ 9 AM

1. Griffiths, 2nd Edition, Problem 8.1 **Use WKB to find the eigen-energies of a potential well with a complicated shape. Algebra advice: square twice to get a linear equation for the eigen-energy.**

2. Griffiths, 2nd Edition, Problem 8.2 **Follow the directions… For the $\hbar^1$ equation, solve for f_1' in terms of p and p', and f_1 will be a logarithm.**

3. Griffiths, 2nd Edition, Problem 7.1 **Variational wavefunctions to estimate ground state energies of the linear and quartic potentials.**

4. Griffiths, 2nd Edition, Problem 7.2 **Variational wavefunction to estimate ground state energy of the harmonic oscillator potential.**

Extra Credit

The Schrödinger equation for the Macroscopic Quantum Wavefunction Ψ(**r**,t) for a superconductor is $i\hbar\frac{\partial\Psi}{\partial t}=\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}-q^*\vec{A}\right)^2\Psi+q^*\phi\Psi$, where $\vec{A}$ is the vector potential, ϕ is the scalar potential, m^* and q^* are the effective mass and charge of a Cooper pair. The macroscopic quantum wavefunction is interpreted as $\Psi(\vec{r},t)=\sqrt{n^*(r,t)}\,e^{i\theta(\vec{r},t)}$, $n^*(\vec{r},t)$ is the local number density and $\theta(\vec{r},t)$ is the space and time-dependent phase.

a) Under the assumption that the number density $n^*(\vec{r},t)=\left|\Psi(\vec{r},t)\right|^2$ is constant in space and time, derive the energy-phase relationship:

$$-\hbar\frac{\partial\theta}{\partial t}=\frac{1}{2n^*}\Lambda J_s^2+q^*\phi$$

from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically. Here the supercurrent density $\vec{J}_s=\frac{1}{\Lambda}(\frac{\hbar}{q^*}\vec{\nabla}\theta-\vec{A})$ and

$$\Lambda=\frac{m^*}{n^*(q^*)^2}.$$

b) Now assume that $n^*(\vec{r},t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:

$$\frac{\partial n^*}{\partial t}=-\vec{\nabla}\bullet(n^*\vec{v}_s)$$

Interpret this result physically (it may help to multiply both sides by q*). Note that the superfluid velocity is given by $\vec{v}_s = \frac{\hbar}{m^*}\vec{\nabla}\theta - \frac{q^*}{m^*}\vec{A}$

Part a: Take the real part of the Schrödinger equation after recognizing the super-current density $\vec{J}_s$ is present in the equation.
Part b: Focus on the imaginary part of the Schrödinger equation.

Physics 402
Spring 2009
Prof. Anlage
Discussion Worksheet for May 6, 2009

1. Variational principle and the 1D harmonic oscillator. Make a guess for the ground state wavefunction of the 1D harmonic oscillator as a simple parabolic function:

$$\psi(x) = \begin{cases} A(a^2 - x^2) & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

As a first step, sketch the potential and wavefunction, and then find the normalization constant *A*.

2. Using the above wavefunction and the length scale 'a' as the variational parameter, find an upper bound for the ground state energy of the 1D harmonic oscillator.

Partial answer: the value of a that minimizes $\langle H \rangle$ is $a^2 = \sqrt{\frac{35}{2}} \frac{\hbar}{m\omega}$