Lecture 33 Highlights

Helium-4 is a Boson. It has two protons and two neutrons in the nucleus, as well as two electrons orbiting the nucleus. All of these particles are spin-1/2. However they pair up into either spin singlet (S=0) or spin triplet (S=1) states, as discussed in section 4.4.3 of Griffiths). Thus it has integer spin. Helium-4 is also a quantum fluid – it will not solidify at any temperature at a pressure of 1 atmosphere. This can be understood qualitatively in terms of the position-momentum uncertainty relation, and the small mass of the He atom. ^{4}He has many unusual macroscopic thermodynamic properties that are governed by quantum mechanics.

Some of these properties include (check out the video),

1) The absence of boiling below the "lambda transition" at T_λ = 2.2 K,
2) Flow with zero viscosity through "superleaks",
3) Finite viscosity when measured by a rotating disk suspended by a torsional oscillator in the fluid,
4) Superfluid film creep,
5) Thermo-mechanical effects, including the fountain effect.

Why is Helium-4 a superfluid? We consider a large collection of ^{4}He atoms in the liquid state inside a box of dimensions $a \times a \times a$. This is a 'gas' of many identical Bosons with overlapping wavefunctions. We know the most likely occupation numbers for the states of such a system:

$$n_s = \frac{g_s}{e^{(E_s-\mu)/k_B T} - 1}$$

The problem now is to determine the states of the system "*s*", the energy levels E_s, the degeneracies of those levels g_s, and finally the appropriate value of the chemical potential μ.

First we take a guess at the sign of the chemical potential μ. Let's assume that the lowest energy state of the system is at energy 0, i.e. $E_1 = 0$. Now suppose $\mu = 0$ as well. This leads to the following result for the most likely occupation number of the ground state: $n_1 = \frac{g_1}{e^{(0-0)/k_B T} - 1} \to \infty$. This does not make sense, since the total number of ^{4}He particles in the box is fixed at N. Now suppose $\mu > 0$. In this case the most likely occupation number for the ground state is $n_1 = \frac{g_1}{e^{-|\mu|/k_B T} - 1} < 0$. In other words, the most likely occupation number for the ground state is negative, which does not make any sense. Hence we are forced to conclude that $\mu < 0$ since only in this case do we get a non-negative and finite occupation number for the ground state of the system. This is an important point that we will return to later.

The next step is to enumerate all of the states available to the system, find their energies and degeneracies, and then enforce the fixed number constraint to determine the chemical potential μ.