Lecture 25 Highlights

A black body radiation spectrum has an energy density (energy per unit volume per unit frequency) given by Planck's formula:

$$\rho(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}, \qquad (1)$$

where ω is the angular frequency of the radiation, k_B is Boltzmann's constant, and T is temperature of the blackbody radiator. On pages 352→354 of Griffiths there is a calculation of the rate of absorption of this type of radiation by a two-level atom (states a and b) assuming random polarization of the light:

$$R_{a\to b} = \frac{\pi e^2 \rho(\omega_{ab})}{3\varepsilon_0 \hbar^2}\left(\left|x_{ab}\right|^2 + \left|y_{ab}\right|^2 + \left|z_{ab}\right|^2\right) \equiv \rho(\omega_{ab}) M_{ab},$$

where $E_b - E_a = \hbar\omega_{ab}$, and the term in parentheses contains the three Cartesian matrix elements between states a and b.

Upon examination of time dependent perturbation theory for the two-state atom, we have seen two processes leading to transitions in the atom. These processes are absorption (atom starts in the lower energy state, absorbs energy from the electromagnetic field, and makes a transition to the upper state), and stimulated emission (atom starts in the upper energy level, the electromagnetic field stimulates a transition to the lower energy state, and energy is added to the electromagnetic field). However if we apply these two processes to equilibrium conditions prevailing in the walls and electromagnetic fields of a "blackbody box," one finds that the rates of absorption and emission of the atoms cannot be balanced. Recognizing this problem, Einstein proposed an additional mechanism of emission, known as <u>spontaneous emission</u>.

We can calculate the spontaneous emission rate A in a blackbody radiator in equilibrium. Let N_a be the number of atoms in the walls of the box in state a, and N_b is the corresponding number in state b. The rate at which atoms join state b (the upper state) is given by a 3-term expression:

$$\frac{dN_b}{dt} = N_a M_{ab}\rho(\omega_{ab}) - N_b A - N_b M_{ba}\rho(\omega_{ab}),$$

where the matrix elements $M_{ba} = M_{ab}$ are symmetric. The terms on the right hand side represent absorption, spontaneous emission, and stimulated emission, respectively. In equilibrium we expect the number of atoms in state b to be unchanging, meaning $\frac{dN_b}{dt} = 0$. Solving for $\rho(\omega_{ab})$ yields:

$$\rho(\omega_{ab}) = \frac{A/M_{ab}}{e^{\hbar\omega_{ab}/k_B T} - 1}. \qquad (2)$$

To get this we assumed a Boltzmann distribution of occupation numbers of states b and a: $\frac{N_b}{N_a} = e^{-(E_b - E_a)/k_B T} = e^{-\hbar\omega_{ab}/k_B T}$. By comparing Eq. (2) to Eq. (1) for the Planck blackbody radiation formula, we can directly determine the spontaneous emission rate:

$$A = \frac{\hbar\omega^3}{\pi^2 c^3}\frac{\pi e^2}{3\varepsilon_0 \hbar^2}\left(\left|x_{ab}\right|^2 + \left|y_{ab}\right|^2 + \left|z_{ab}\right|^2\right).$$

The spontaneous emission rate increases quickly with increasing energy difference between upper and lower states (ω^3). This limits the operation of lasers at high frequencies (beyond the optical range).

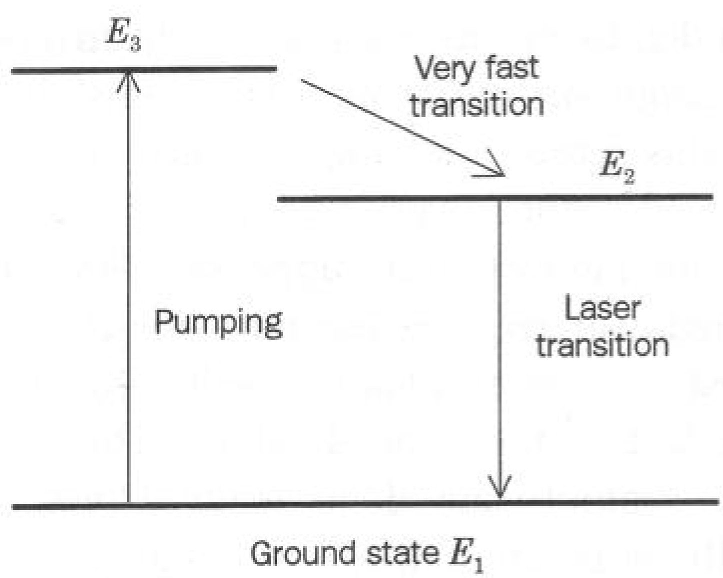
A LASER creates Light Amplification by Stimulated Emission of Radiation. It is basically a Xerox machine for photons. Three requirements must be met:

1) A pair of energy levels are needed for which a transition can take place.

2) A population inversion of the two states must be created. In other words a non-equilibrium situation in which there are more atoms in the upper state than the lower state must be created.

3) A resonance is required to keep the photons passing back and forth through the gain medium to allow the "Xeroxing" to take place by means of stimulated emission.

A typical laser uses 3 levels, as shown in the diagram:



Typically the pumping transition from E_1 to E_3 involves a $\Delta\ell = +1$ transition. The "very fast transition" shown in the diagram from E_3 to E_2 involves another $\Delta\ell = +1$ transition. The transition back to the ground state from E_2 to E_1 now involves a $\Delta\ell = -2$ transition which is dipole forbidden. Hence a population inversion will be created in energy level E_2. An atom in this state may undergo a spontaneous emission through a quadrupole process. This photon, if it stays inside the laser cavity, will then create other identical (in-phase) photons through stimulated emission. These photons will create other identical prodigy and there will be a chain reaction of identical photons built up inside the laser. This is the source of the intense coherent (phase coherent) light coming out of a laser.