

Lecture 23 Highlights

The phenomenon of Rabi oscillations is very different from our everyday experience of how macroscopic objects (i.e. those made up of many atoms) absorb electromagnetic radiation. When illuminated with light (like French fries at McDonalds) objects tend to steadily absorb the light and heat up to a steady state temperature, showing no signs of oscillation with time. We did a calculation of the transition probability of an atom illuminated with a broad spectrum of light and found that the probability increased linearly with time, leading to a constant transition rate. This difference comes about because the many absorption processes at different frequencies produce a ‘smeared-out’ response that destroys the coherent Rabi oscillations and results in a classical incoherent absorption.

Selection Rules

All of the transition probabilities are proportional to “dipole matrix elements,” such as x_{jn} above. These are similar to dipole moment calculations for a charge distribution in classical physics, except that they involve the charge distribution in two different states, connected by the dipole operator for the transition. In many cases these integrals are zero because of symmetries of the associated wavefunctions. This gives rise to selection rules for possible transitions under the dipole approximation (atom size much smaller than the electromagnetic wave wavelength).

For example, consider the matrix element $z_{n,\ell,m;n',\ell',m'}$ between states of the Hydrogen atom labeled by the quantum numbers n, ℓ, m and n', ℓ', m' :

$$z_{n,\ell,m;n',\ell',m'} = \iiint \psi_{n,\ell,m}(\vec{r}) z \psi_{n',\ell',m'}(\vec{r}) d^3r$$

To make progress, first consider just the ϕ integral. In this case, since $z = r \cos \theta$, the ϕ dependence comes entirely from the spherical harmonics in the Hydrogen atom wave-functions:

$$z_{n,\ell,m;n',\ell',m'} \propto \int_0^{2\pi} e^{-im\phi} e^{+im'\phi} d\phi$$

Since m and m' are integers, this integral will be zero unless $m = m'$. This gives rise to a selection rule from this type of matrix element, namely $\Delta m = 0$, to get a non-zero transition probability.

After considering the ϕ integral for the other polarization directions, x and y (which are proportional to $\cos \phi$ and $\sin \phi$), one finds another possible selection rule: $\Delta m = \pm 1$. Note that any general polarization direction can be written as a linear superposition of x, y, and z-directed electric fields. By exploring all of these possibilities, we eliminate any bias or prejudice implied by our arbitrary choice of a coordinate system to describe the physics of atom-light interaction.

Examining the θ integrals, one finds that the Hydrogen atom wavefunctions are proportional to the associated Legendre polynomials $P_\ell^{|m|}(\cos \theta)$, while the dipole operators x, y, and z bring in factors of $\sin \theta$ and $\cos \theta$ in the matrix element integrals. One can use the identities:

$$\cos\theta \times P_\ell^{|m|}(\cos\theta) = \frac{(\ell - |m| + 1)P_{\ell+1}^{|m|}(\cos\theta) + (\ell + |m|)P_{\ell-1}^{|m|}(\cos\theta)}{2\ell + 1}$$

$$\sin\theta \times P_\ell^{|m|}(\cos\theta) = \frac{P_{\ell+1}^{|m+1|}(\cos\theta) - P_{\ell-1}^{|m+1|}(\cos\theta)}{2\ell + 1}.$$

For these identities, see the site <http://functions.wolfram.com/Polynomials/LegendreP2/17/02/01/>.

Using the orthogonality of the Associated Legendre functions, one comes up with a new set of selection rules on the angular momentum quantum number: $\Delta\ell = \pm 1$.

Since both integrals (ϕ and θ) are done in sequence, both sets of selection rules must be satisfied to get a non-zero probability of transition. Thus the dipole approximation selection rules are: $\Delta m = 0$ or ± 1 and $\Delta\ell = \pm 1$.