Phys 402
Spring 2009
Homework 10
Due Friday, May 8, 2009 @ 9 AM

1. Griffiths, 2nd Edition, Problem 8.1

2. Griffiths, 2nd Edition, Problem 8.2

3. Griffiths, 2nd Edition, Problem 7.1

4. Griffiths, 2nd Edition, Problem 7.2

Extra Credit

The Schrödinger equation for the Macroscopic Quantum Wavefunction Ψ(**r**,t) for a superconductor is $i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} - q^*\vec{A}\right)^2 \Psi + q^*\phi\Psi$, where $\vec{A}$ is the vector potential, ϕ is the scalar potential, m^* and q^* are the effective mass and charge of a Cooper pair. The macroscopic quantum wavefunction is interpreted as $\Psi(\vec{r},t) = \sqrt{n^*(r,t)}\, e^{i\theta(\vec{r},t)}$, $n^*(\vec{r},t)$ is the local number density and $\theta(\vec{r},t)$ is the space and time-dependent phase.

a) Under the assumption that the number density $n^*(\vec{r},t) = \left|\Psi(\vec{r},t)\right|^2$ is constant in space and time, derive the energy-phase relationship:

$$-\hbar\frac{\partial \theta}{\partial t} = \frac{1}{2n^*}\Lambda J_s^2 + q^*\phi$$

from the real part of the macroscopic quantum Schrödinger equation. Interpret this equation physically. Here the supercurrent density $\vec{J}_s = \frac{1}{\Lambda}(\frac{\hbar}{q^*}\vec{\nabla}\theta - \vec{A})$ and

$$\Lambda = \frac{m^*}{n^*(q^*)^2}.$$

b) Now assume that $n^*(\vec{r},t)$ is NOT constant in either space or time. Show that the imaginary part of the macroscopic Schrödinger equation yields:

$$\frac{\partial n^*}{\partial t} = -\vec{\nabla}\bullet(n^*\vec{v}_s)$$

Interpret this result physically (it may help to multiply both sides by q*). Note that the superfluid velocity is given by $\vec{v}_s = \frac{\hbar}{m^*}\vec{\nabla}\theta - \frac{q^*}{m^*}\vec{A}$