Special Relativity Questions on the Physics GRE subject test

23. Two spaceships approach Earth with equal speeds, as measured by an observer on Earth, but from opposite directions. A meterstick on one spaceship is measured to be 60 cm long by an occupant of the other spaceship. What is the speed of each spaceship, as measured by the observer on Earth?

(A) $0.4c$
(B) $0.5c$
(C) $0.6c$
(D) $0.7c$
(E) $0.8c$

24. A meter stick with a speed of $0.8c$ moves past an observer. In the observer's reference frame, how long does it take the stick to pass the observer?

(A) 1.6 ns
(B) 2.5 ns
(C) 4.2 ns
(D) 6.9 ns
(E) 8.3 ns

39. A beam of muons travels through the laboratory with speed $v = \frac{4}{5}c$. The lifetime of a muon in its rest frame is $\tau = 2.2 \times 10^{-6}$ s. The mean distance traveled by the muons in the laboratory frame is

(A) 530 m
(B) 660 m
(C) 880 m
(D) 1,100 m
(E) 1,500 m

55. A distant galaxy is observed to have its hydrogen-β line shifted to a wavelength of 580 nm, away from the laboratory value of 434 nm. Which of the following gives the approximate velocity of recession of the distant galaxy? (Note: $\frac{580}{434} \approx \frac{4}{3}$)

(A) $0.28c$
(B) $0.53c$
(C) $0.56c$
(D) $0.75c$
(E) $0.86c$

94. An observer O at rest midway between two sources of light at $x = 0$ and $x = 10$ m observes the two sources to flash simultaneously. According to a second observer O', moving at a constant speed parallel to the x-axis, one source of light flashes 13 ns before the other. Which of the following gives the speed of O' relative to O?

(A) $0.13c$
(B) $0.15c$
(C) $0.36c$
(D) $0.53c$
(E) $0.62c$

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$\ell = V\,\Delta t$ This is how one measures length by means of a time interval

We know V = 0.8c

ℓ is the length contracted result: $\ell = 1$ meter / γ

$$\gamma = \frac{1}{\sqrt{1-(V/c)^2}} = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{1}{\sqrt{9/25}} = \frac{5}{3}$$

Hence $$\Delta t = \frac{\ell}{V} = \frac{1\,m\ \times 3/5}{\left(\frac{4}{5}\right)c} = \frac{3}{4}\frac{1\,m}{c} = 2.5\,ns$$ The answer is "B"

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The lifetime is the proper lifetime τ, measured in the rest frame of the muon

As measured in the lab frame, the lifetime of the muon is dilated: $\tau_{Lab} = \gamma\tau$

In this case: $$\gamma = \frac{1}{\sqrt{1-(V/c)^2}} = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{1}{\sqrt{9/25}} = \frac{5}{3}$$

Hence: $$\tau_{Lab} = \gamma\tau = \frac{5}{3} \times 2.2\ \mu s = 3.67 \mu s$$

At the observed speed, the muons will travel a distance (on average) of: $\Delta x_{Lab} = \tau_{Lab} \times V \cong 880\ m$

The answer is “C”

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Remember the red-shift formula: $\frac{f_0}{f} = \sqrt{\frac{1+\beta}{1-\beta}}$

But in this case we are given wavelengths rather than frequency. Use $f = c/\lambda$

Hence: $\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}}$

In this case we are given that : $\frac{\lambda}{\lambda_0} = \frac{4}{3}$

Solving for β we find: $\beta = \frac{\left(\frac{\lambda}{\lambda_0}\right)^2 - 1}{\left(\frac{\lambda}{\lambda_0}\right)^2 + 1} = \frac{\frac{16}{9} - 1}{\frac{16}{9} + 1} = \frac{7}{25} = \frac{28}{100}$

The answer is "A"

Note the attempt to confuse you with the "hydrogen-β line"!

94. An observer O at rest midway between two sources of light at $x = 0$ and $x = 10$ m observes the two sources to flash simultaneously. According to a second observer O', moving at a constant speed parallel to the x-axis, one source of light flashes 13 ns before the other. Which of the following gives the speed of O' relative to O?

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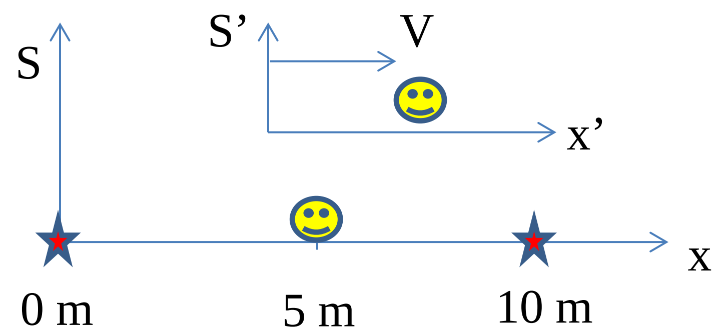

The two flashes arrive simultaneously at the observer in S, but $\Delta t' = 13\ ns$ in S'

Use the Lorentz transformation for time: $t' = \gamma(t - xV/c^2)$

In this case: $t_2' - t_1' = \Delta t' = \gamma\left(t_2 - \frac{x_2 V}{c^2} - t_1 + \frac{x_1 V}{c^2}\right)$

But: $t_1 = t_2 = 0$ so that $\Delta t' = \gamma \frac{V}{c^2} 10\ m$ We want to find V, or equivalently β

Note that $\gamma \frac{V}{c^2} = \gamma\beta/c$ and one can find that $\gamma^2\beta^2 = \frac{\beta^2}{1-\beta^2}$

After some algebra, one finds: $\beta = \frac{x}{\sqrt{1+x^2}}$ with $x = \frac{c\Delta t\prime}{10\ meters} = 0.39$ (dimension-less)

Finally: $\beta = \frac{0.39}{\sqrt{1+0.39^2}} = 0.36$

The answer is “C”

This is the 3rd hardest question on the exam

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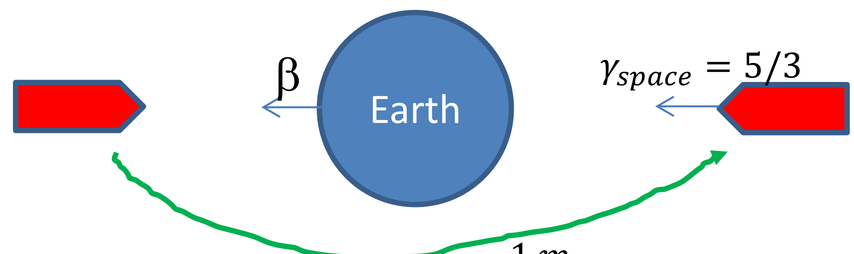
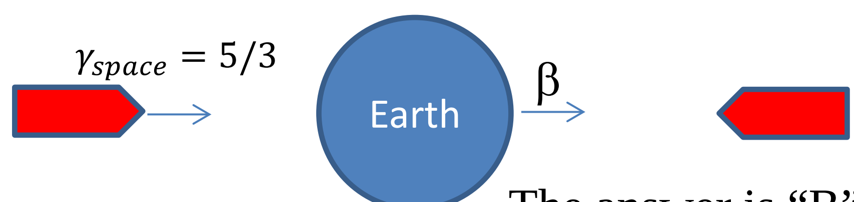
From Earth's IRF:

From the left spaceship's (LSS) IRF:



$$\ell = \frac{1\ m}{\gamma_{space}} = 0.6\ m$$

From the right spaceship's (RSS) IRF:



From the observed length contraction we know that the two spacecraft approach each other such that $\gamma_{space} = 5/3$. This corresponds to $\beta_{space} = 4/5$

The LSS observes earth approaching at speed βc

The RSS observes earth approaching at speed βc

v_x is the speed of earth in the LSS IRF

v_x' is the speed of earth in the RSS IRF

In the LSS IRF: $v_x = -\beta c$

In the RSS IRF: $v_x' = +\beta c$

These two IRFs are moving at relative speed V = (4/5)c

Use the Lorentz velocity transformation law to translate the Earth's velocity between the two IRFs:

$$v_x = \frac{v_x' + V}{1 + V v_x'/c^2}$$

Solve for β and you get $\beta = -1/2$

The answer is "B"