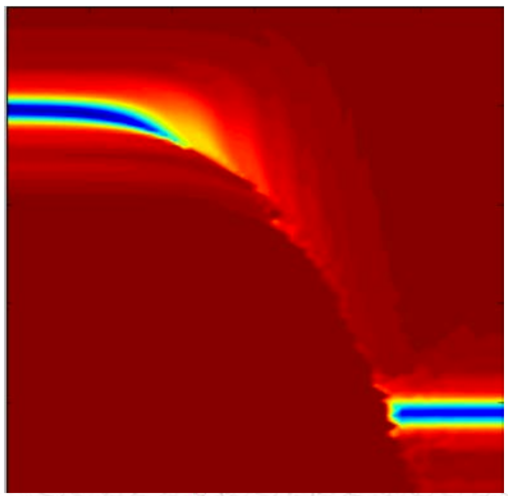
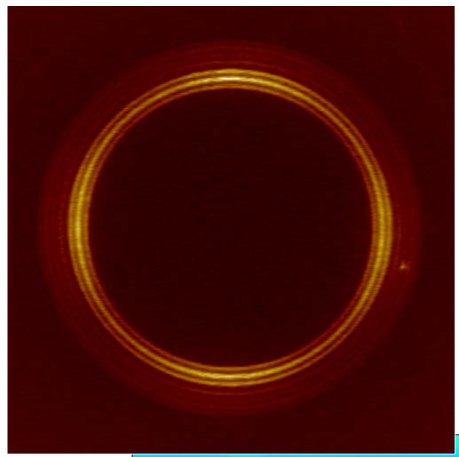
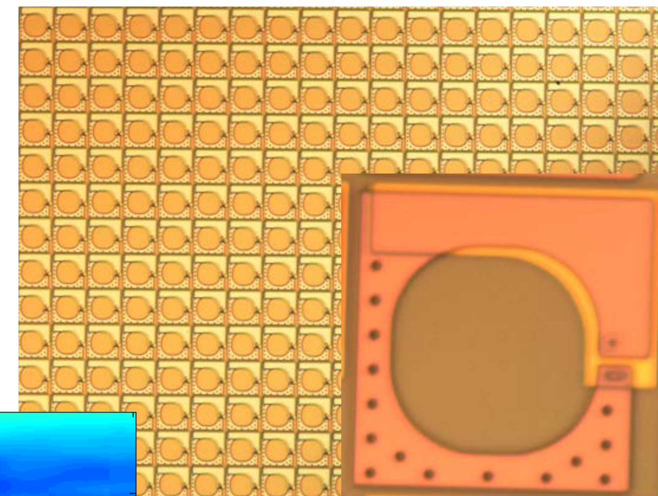
Welcome to PHYS 276!!

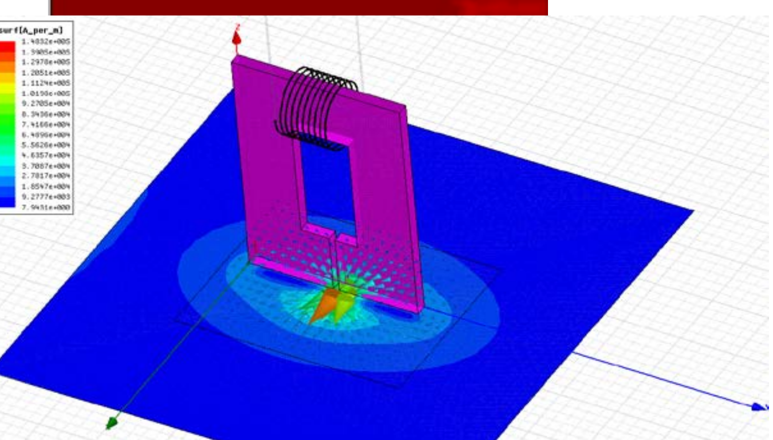
Instructor: Professor Steven Anlage

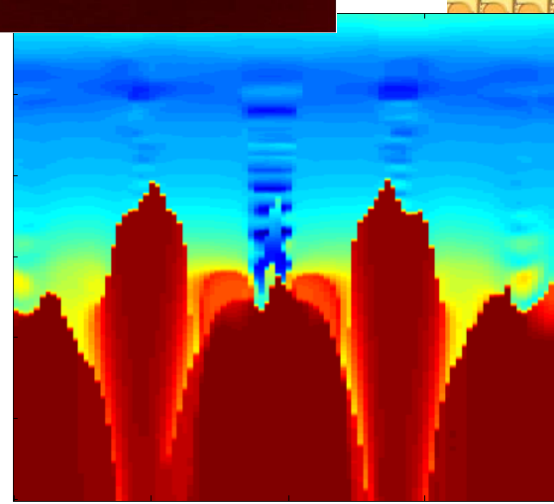
- Research Specialty: Experimental Condensed Matter Physics
- http://anlage.umd.edu/AnlageHome.htm

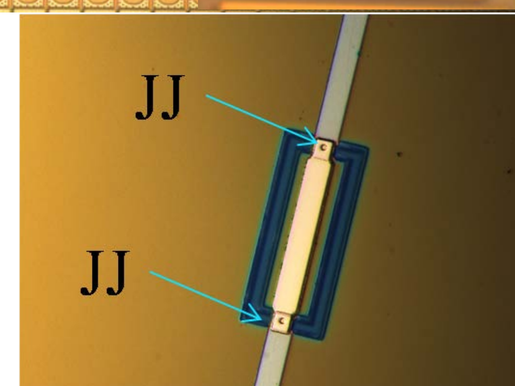












Teaching Assistants

TA Introduction: Rui Zhang (Wednesday) (ryez@umd.edu)
Iakov Boyko (Thursday) (iboyko@umd.edu)

Introductions

Name, class, major?

Goals for this Course

- Learn experimental techniques and equipment for studying electricity and magnetism
- Reinforce understanding of E&M and electronics gained last semester in lecture course through hands-on experience
- Learn importance of proper record keeping and scientific writing for experimental science: learn how to write a lab report
- Further develop skills in error analysis, beyond those gained in 174, 275

Class web page: http://www.physics.umd.edu/courses/Phys276/AnlageSpring15/

Upload all assignments on ELMS

Schedule

You will do all labs (except the one today) with an assigned partner. See the class website for the assignment.

TENTATIVE SCHEDULE FOR PHYSICS 276, SPRING 2015, Prof. Anlage

	Date	Mtg.#	Topics	Lab Report Due
Week 1				
	1/28/2015; 1/29/2015	1	Experiment 0 - Introduction, Review, and Diagnostics	Spreadsheet
Week 2				
	2/4/2015; 2/5/2015	2	Experiment 1 - Input and Output Impedance	Spreadsheet
Week 3				
	2/11/2015; 2/12/2015	3	NO LAB	
Week 4				
	2/18/2015; 2/19/2015	4	Experiment 2 - Diodes	Spreadsheet and Full Lab Report
Week 5				
	2/25/2015; 2/26/2015	5	Experiment 3 - Capacitors and Filters	Spreadsheet
Week 6				
	3/4/2015; 3/5/2015	6	Experiment 4 - Induction	Spreadsheet
Week 7				
	3/11/2015; 3/12/2015	7	Oral Presentations (No Lab)	
Week 8				
	3/18/2015; 3/19/2015	8	Spring Break	
Week 9				
	3/25/2015; 3/26/2015	9	Experiment 5 - Filters	Spreadsheet
Week 10				
	4/1/2015; 4/2/2015	10	Experiment 6 - Radio	Spreadsheet and Full Lab Report
Week 11				
	4/8/2015; 4/9/2015	11	Experiment 7 - Crystal Radio	Spreadsheet
Week 12				
	4/15/2015; 4/16/2015	12	Experiment 8 - Transistor and Amplifier	Spreadsheet
Week 13				
	4/22/2015; 4/23/2015	13	Experiment 9 - Transistor Radio	Spreadsheet and Full Lab Report
Week 14				
	4/29/2015; 4/30/2015	14	Makeup Lab / Final Exam Practice	
Week 15				
	5/6/2015; 5/7/2015	15	Final Exam	

What we will be doing
(and you will be graded on)

- Prelabs
- In-class spreadsheet
- Formal lab reports
- Oral presentation
- Final lab-practical exam

Syllabus

Our contract: let’s go through it.

ELMS

Will use to turn in our pre-labs, in-class spreadsheets, our Lab reports, and the slides for the oral presentations.

Goals

- Learn experimental techniques and equipment for studying electricity and magnetism
- Reinforce understanding of E&M and electronics gained from lecture course through hands-on experience

Lab 0

- Review error analysis and oscilloscope

No lab report, just upload excel spreadsheet

Goals

• learn importance of proper recording keeping and scientific writing for experimental science: learn how to write a lab report

A Journal Publication

PHYSICAL REVIEW D, VOLUME 61, 072001

Extraction of the width of the W boson from measurements of $\sigma(p\bar{p}\to W+X)\times B(W\to e\nu)$ and $\sigma(p\bar{p}\to Z+X)\times B(Z\to ee)$ and their ratio

B. Abbott,[45] M. Abolins,[42] V. Abramov,[18] B. S. Acharya,[11] I. Adam,[44] D. L. Adams,[54] M. Adams,[28] S. Ahn,[27] V. Akimov,[16] G. A. Alves,[2] N. Amos,[41] E. W. Anderson,[34] M. M. Baarmand,[47] V. V. Babintsev,[18] L. Babukhadia,[20] A. Baden,[38] B. Baldin,[27] S. Banerjee,[11] J. Bantly,[51] E. Barberis,[21] P. Baringer,[35] J. F. Bartlett,[27] A. Belyaev,[17] S. B. Beri,[9] I. Bertram,[19] V. A. Bezzubov,[18] P. C. Bhat,[27] V. Bhatnagar,[9] M. Bhattacharjee,[47] G. Blazey,[29] S. Blessing,[25] P. Bloom,[22] A. Boehnlein,[27] N. I. Bojko,[18] F. Borcherding,[27] C. Boswell,[24] A. Brandt,[27] R. Breedon,[22] G. Briskin,[51] R. Brock,[42] A. Bross,[27] D. Buchholz,[30] V. S. Burtovoi,[18] J. M. Butler,[39] W. Carvalho,[2] D. Casey,[42] Z. Casilum,[47] H. Castilla-Valdez,[14] D. Chakraborty,[47] S. V. Chekulaev,[18] W. Chen,[47] S. Choi,[13] S. Chopra,[25] B. C. Choudhary,[24] J. H. Christenson,[27] M. Chung,[28] D. Claes,[43] A. R. Clark,[21] W. G. Cobau,[38] J. Cochran,[24] L. Coney,[32] W. E. Cooper,[27] D. Coppage,[35] C. Cretsinger,[46] D. Cullen-Vidal,[51] M. A. C. Cummings,[29] D. Cutts,[51] O. I. Dahl,[21] K. Davis,[20] K. De,[52] K. Del Signore,[41] M. Demarteau,[27] D. Denisov,[27] S. P. Denisov,[18] H. T. Diehl,[27] M. Diesburg,[27] G. Di Loreto,[42] P. Draper,[52] Y. Ducros,[8] L. V. Dudko,[17] S. R. Dugad,[11] A. Dyshkant,[18] D. Edmunds,[42] J. Ellison,[24] V. D. Elvira,[47] R. Engelmann,[47] S. Eno,[38] G. Eppley,[54] P. Ermolov,[17] O. V. Eroshin,[18] H. Evans,[44] V. N. Evdokimov,[18] T. Fahland,[23] M. K. Fatyga,[46] S. Feher,[27] D. Fein,[20] T. Ferbel,[46] H. E. Fisk,[27] Y. Fisyak,[48] E. Flattum,[27] G. E. Forden,[20] M. Fortner,[29] K. C. Frame,[42] S. Fuess,[27] E. Gallas,[27] A. N. Galyaev,[18] P. Gartung,[24] V. Gavrilov,[16] T. L. Geld,[42] R. J. Genik II,[42] K. Genser,[27] C. E. Gerber,[27] Y. Gershtein,[51] B. Gibbard,[48] B. Gobbi,[30] B. Gómez,[5] G. Gómez,[38] P. I. Goncharov,[18] J. L. González Solís,[14] H. 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(DØ Collaboration)

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Things to note:

Introduction
Description of the apparatus
Figure captions
Table captions
Tables with data and estimated errors
Discussion of errors
Conclusion

http://www2.physics.umd.edu/~eno/teaching/276/f09/lectures/example_publication.pdf

Lab Reports

See "Guidelines for Written Lab Reports" and

"Grading Rubric for Lab Reports" on class web page

You will do 3 full lab reports this semester

Goals

• further develop skills in error analysis, beyond that gained in 174, 275

Please obtain a standard book on error analysis, such as:
Introduction to Error Analysis, J. Taylor, University Science Books, 1997

Estimating Errors: Review

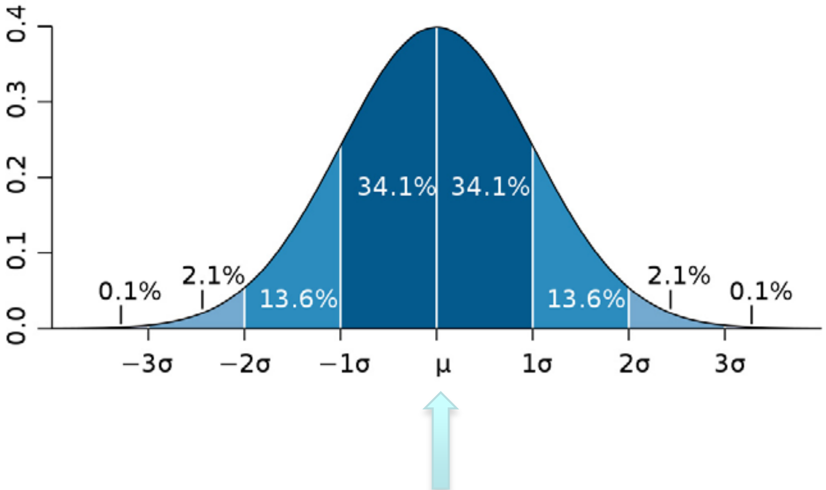
- Systematic errors : sources of error that have the same size effect on every measurement that is made (or a correlated effect)
    - a ruler that was not manufactured correctly
    - a consistently delayed reaction when using a stop watch
    - your inability to perfectly estimate the size of a stray magnetic field from your computer that leaks into your experimental area
- Random errors : sources of error whose effect varies with each measurement
    - precision of your measuring device
    - when using a stop watch, a reaction time that sometimes anticipates the event, some times is in retard of the event.
- Total error: is the quadrature sum of the random and systematic errors.

$$\sigma_{total} = \sqrt{\sigma^2_{random} + \sigma^2_{systematic}}$$

How many digits should we quote on an error, and why?

Random Errors

Usually distributed according to a Gaussian Distribution



True value

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

68% of data within 1 “sigma”
95% within 2 “sigma” (σ)
True value is most probable value

What were some random errors from 174?

How did we estimate them?

Systematic Errors

Systematic errors come from your limits on your ability to asses the accuracy of the device, even when it is being used correctly.

During this class, they will be estimated using information from the manufacturer of the measuring device.

Systematic Errors are “known unknowns”. They are not mistakes. Do not use data that has known mistakes. If you have made a mistake, you need to correct the data or retake the data. For example, failing to take into account the resistance of your ammeter when testing ohm’s law is a mistake, not a systematic error. Uncertainties on its resistance, because your ability to measure its value is limited due to uncertainties in the calibration of the device, do lead to a systematic error.

Error Propagation: Review

You have taken a measurement which has a random error (uncertainty), and want to use it in a calculation. What is the uncertainty on the result of the calculation due to the uncertainty on the measurement?

$$y = f(x_1, x_2, x_3, \ldots)$$

$$\sigma_y = \sqrt{\sum_i \left(\frac{\partial y}{\partial x_i}\sigma_{x_i}\right)^2}$$

Note: this formula is for the uncertainty on the result of a simple, algebraic calculation. Errors on fits need to be handled in a different way.

Error Propogation: Example

Length of a table is 2 m ± 0.01 m

Width is 1 m ± 0.005 m

What is the area? What is the error on the area?

$$A = L \times W$$

$$\frac{\partial A}{\partial L} = W$$

$$\frac{\partial A}{\partial W} = L$$

$$\sigma_A = \sqrt{(W \times \sigma_L)^2 + (L \times \sigma_W)^2} = A\sqrt{(\sigma_L/L)^2 + (\sigma_W/W)^2}$$

$$= 2\ m^2\sqrt{(0.01/2)^2 + (0.005/1)^2} = 2\ m^2\sqrt{0.005^2 + 0.005^2}$$

How to set this up in your spreadsheet to minimize the chance of an algebraic mistake?
When you are done how to check if it makes sense?

Error Propagation: Example

You take 3 independent measurements of the period of a pendulum. You get 15±0.1s, 14.8±0.1s, and 14.9±0.1s. What is the average of these 3 measurements?

$$\langle T \rangle = \frac{x_1 + x_2 + x_3}{3} = 14.9 \text{ s}$$

$$\sigma_{\langle T \rangle} = \sqrt{\left(\frac{\partial \langle T \rangle}{\partial x_1}\sigma_{x_1}\right)^2 + \left(\frac{\partial \langle T \rangle}{\partial x_2}\sigma_{x_2}\right)^2 + \left(\frac{\partial \langle T \rangle}{\partial x_3}\sigma_{x_3}\right)^2}$$

$$\frac{\partial \langle T \rangle}{\partial x_1} = \frac{1}{3}$$

$$\sigma_{\langle T \rangle} = \sqrt{\left(\frac{\sigma_{x_1}}{3}\right)^2 + \left(\frac{\sigma_{x_2}}{3}\right)^2 + \left(\frac{\sigma_{x_3}}{3}\right)^2}$$

$$= \frac{1}{\sqrt{3}}\sigma = 0.03 \text{ s}$$

Error Propagation: Try it

Calculate this in EXCEL. You will submit your work at the end of class. We'll move on when all of you are done.

You drop a ball (initially at rest) and it falls 3 m ± 0.01 m in 0.785 ± 0.002 s.

What is the acceleration due to gravity *g*?

Strongly suggest you put each number in a separate cell, the formula for each partial derivative in a cell, and then multiply, square and add them up. It makes it easier to spot errors!

answer: $9.737 \pm 0.059 \ \text{m/s}^2$

Review: Chi^2

You've made a measurement and want to compare it to theory. How do you do this?

$$\chi^2 = \sum_{data} \frac{(data - theory)^2}{error^2}$$

How far is the data from the theory in natural units (size of the error)?

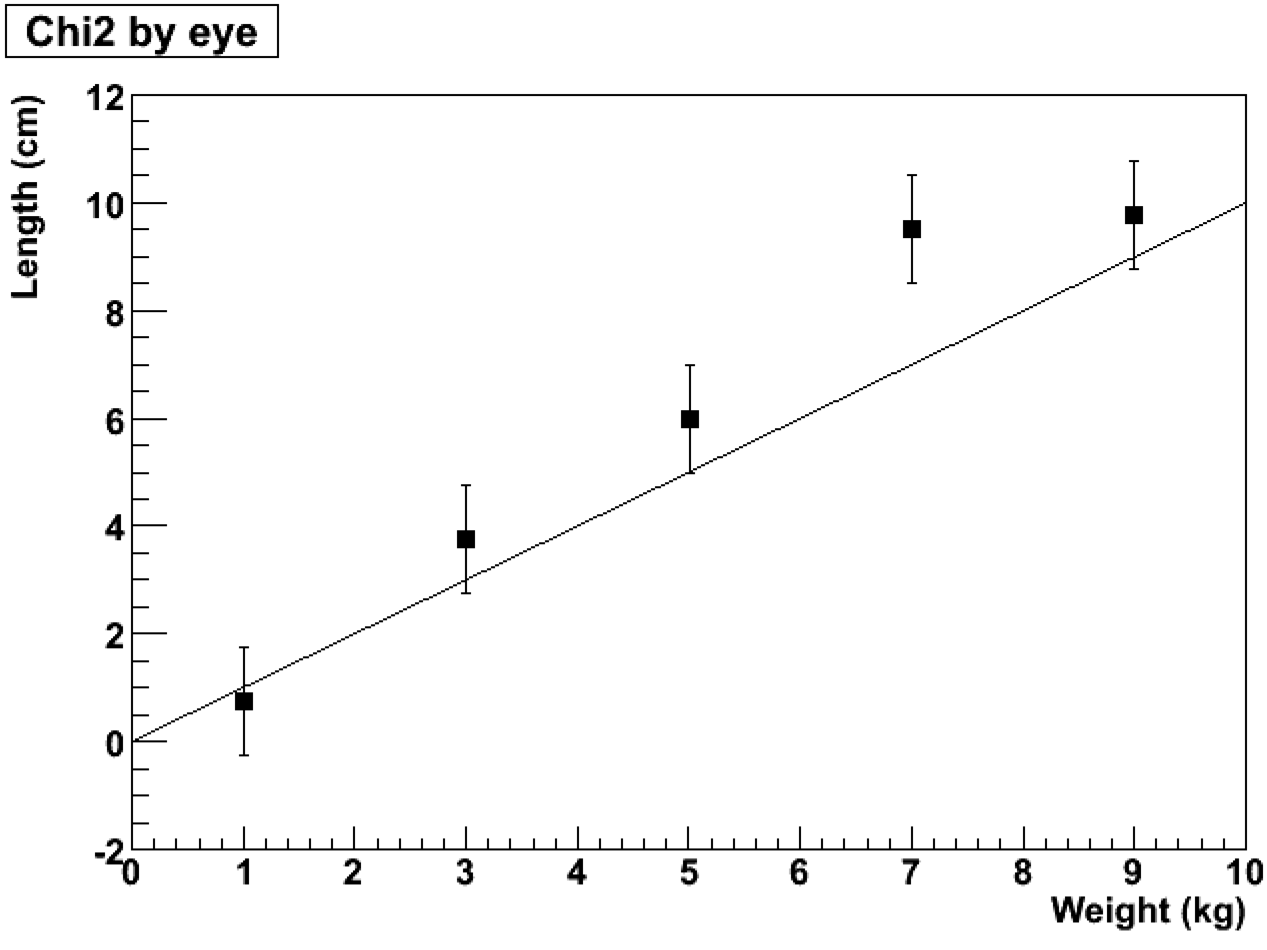
If the data is in good agreement with the theory, what should the value of chi^2 be?

ν: Degrees of freedom: number of data points – number of parameters in the theory that are determined using the data

Reduced chi2: chi2/ν

Review: Chi2

Estimating the chi2 for this data to the theory curve by eye (no fitting parameters). Put the result in your spreadsheet.



Values of χ^2 that Indicate Significant Disagreement Between Theory and Experiment

Why are parts of this table in yellow?

Using this table, find the probability that the graph on the previous slide would have a chi^2 that big or bigger from random errors alone. Put the result in your spreadsheet.

	$P(\chi^2,v)$									
v	0.99	0.95	0.90	0.70	0.50	0.30	0.10	0.05	0.01	0.001
1	0.0002	0.004	0.052	0.15	0.46	1.07	2.7	3.8	6.6	11
2	0.020	0.103	0.210	0.71	1.39	2.41	4.6	6.0	9.2	14
3	0.11	0.35	0.59	1.43	2.37	3.67	6.3	7.8	11.3	16
4	0.30	0.71	1.06	2.20	3.36	4.88	7.8	9.5	13.3	18
5	0.56	1.1	1.6	3.0	4.4	6.1	9.2	11.1	15.1	21
6	0.87	1.6	2.2	3.8	5.3	7.2	10.6	12.6	16.8	22
7	1.2	2.2	2.8	4.7	6.3	8.4	12.0	14.1	18.5	24
8	1.6	2.7	3.5	5.5	7.3	9.5	13.4	15.5	20	26
9	2.1	3.3	4.2	6.4	8.3	10.7	15	17	22	28
10	2.6	3.9	4.9	7.3	9.3	11.8	16	18	23	30
12	3.6	5.2	6.3	9.0	11	14	19	21	26	33
15	5	7	9	12	14	17	22	25	31	38
20	8	11	12	16	19	23	28	31	38	45
30	15	18	21	26	29	34	40	44	51	60
50	30	35	38	44	49	52	63	68	76	87

$P(\chi^2,v)$ TOO BIG	$P(\chi^2,v)$ OK	$P(\chi^2,v)$ TOO SMALL
χ^2 TOO SMALL	χ^2 OK	χ^2 TOO BIG
"given the error bars, the theory is much too close to the experiment"	"theory and experiment are consistent"	"given the error bars, there is a significant disagreement between theory and experiment"
BAD	GOOD	BAD

Or use the chidist function in EXCEL

What to do if your chi^2 turns out to be too low or too high?

What is too low or too high?
What could the causes be?
What to do in your lab report?

Fitting: Review

You have some data points. What straight line curve best “fits” the data? -> what values of slope *m* and intercept *b* minimize the chi^2 between the line and the data.

$z=y-(mx+b)$

“perfect fit” z would be zero. Theory is that z is zero. Have 2 fitting parameters. *m* and *b*. Measurements are x and y

$$\sigma_z = \sqrt{(\frac{\partial z}{\partial y}\sigma_y)^2 + (\frac{\partial z}{\partial x}\sigma_x)^2}$$

$$= \sqrt{(\sigma_y)^2 + (m\sigma_x)^2}$$

$$\chi^2 = \sum_{data} \frac{(z-0)^2}{\sigma^2{}_z} = \sum_{data} \frac{(y-mx-b)^2}{(\sigma_y)^2 + (m\sigma_x)^2}$$

If ignore the errors on x.

$$\chi^2 = \sum_{data} \frac{(y-mx-b)^2}{(\sigma_y)^2}$$

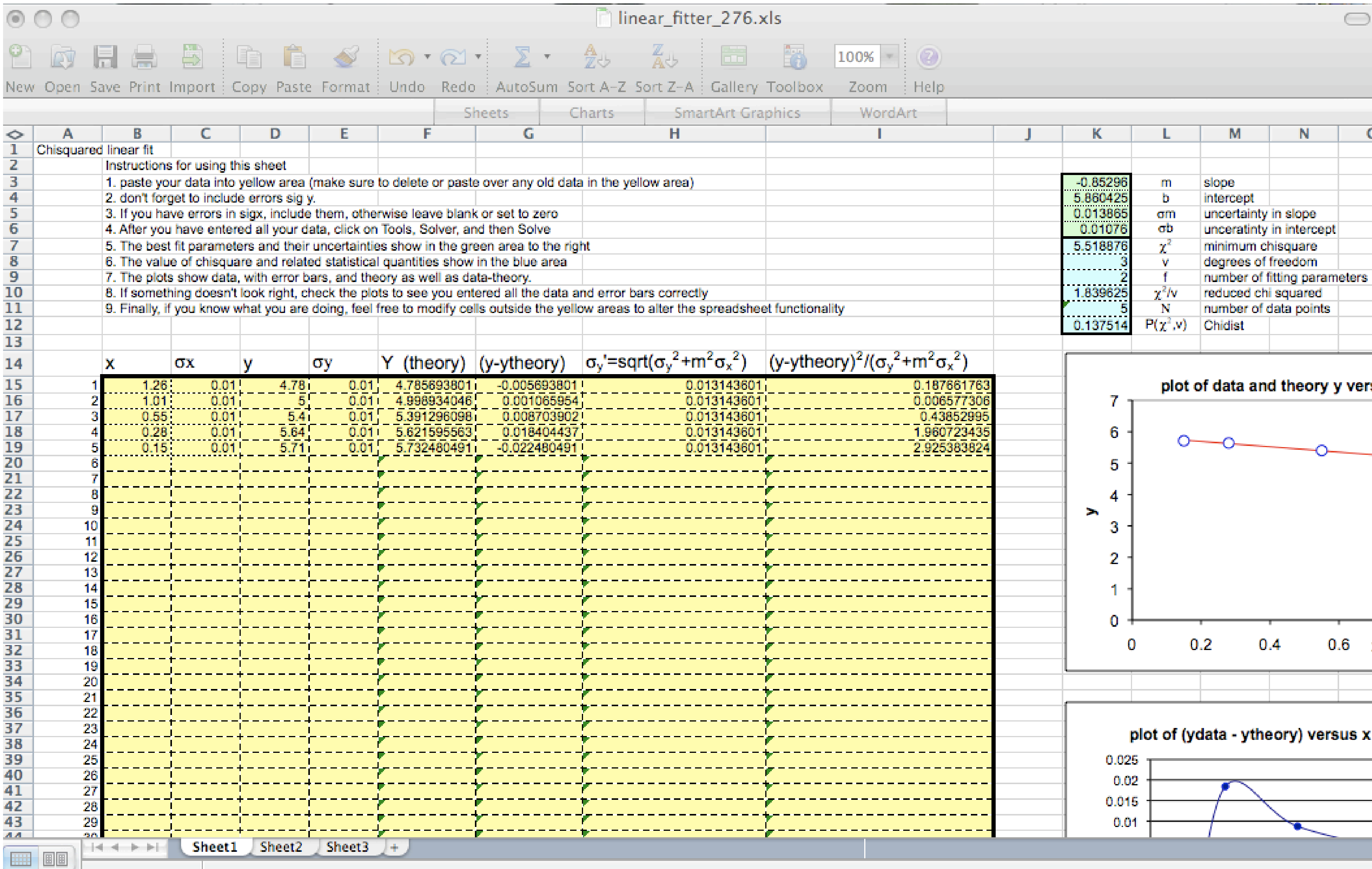
Practice linear fitting

Make a plot of this data including error bars.

Time (s)	Position (cm)
1+-0.1	2+-2
2+-0.1	11+-2
3+-0.1	13+-2
4+-0.1	20+-2
5+-0.1	28+-2
6+-0.1	31+-2

linear_fitter_276.xls

When copying this into your spreadsheet, it is highly recommended to copy the entire sheet, and make it a separate new sheet in your spreadsheet.


Chisquared linear fit

Instructions for using this sheet

1. paste your data into yellow area (make sure to delete or paste over any old data in the yellow area)
2. don't forget to include errors sig y.
3. If you have errors in sigx, include them, otherwise leave blank or set to zero
4. After you have entered all your data, click on Tools, Solver, and then Solve
5. The best fit parameters and their uncertainties show in the green area to the right
6. The value of chisquare and related statistical quantities show in the blue area
7. The plots show data, with error bars, and theory as well as data-theory.
8. If something doesn't look right, check the plots to see you entered all the data and error bars correctly
9. Finally, if you know what you are doing, feel free to modify cells outside the yellow areas to alter the spreadsheet functionality

-0.85296	m	slope
5.860425	b	intercept
0.013865	σm	uncertainty in slope
0.01076	σb	unceratinty in intercept
5.518876	χ^2	minimum chisquare
3	ν	degrees of freedom
2	f	number of fitting parameters
1.839625	χ^2/ν	reduced chi squared
5	N	number of data points
0.137514	$P(\chi^2,\nu)$	Chidist

	x	σx	y	σy	Y (theory)	(y-ytheory)	$\sigma_y'=\mathrm{sqrt}(\sigma_y^2+m^2\sigma_x^2)$	$(y-\mathrm{ytheory})^2/(\sigma_y^2+m^2\sigma_x^2)$
1	1.26	0.01	4.78	0.01	4.785693801	-0.005693801	0.013143601	0.187661763
2	1.01	0.01	5	0.01	4.998934046	0.001065954	0.013143601	0.006577306
3	0.55	0.01	5.4	0.01	5.391296098	0.008703902	0.013143601	0.43852995
4	0.28	0.01	5.64	0.01	5.621595563	0.018404437	0.013143601	1.960723435
5	0.15	0.01	5.71	0.01	5.732480491	-0.022480491	0.013143601	2.925383824
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Practice: Linear Fit

Use linear_fitter_276.xls to fit this data. Calculate the chi^2 and calculate the probability to get a chi2 this big or bigger due to random errors.

Linear fits

You know how to use the solver to minimize the chi^2 to do linear fits…

Where do the errors on the slope and intercept come from?

During this lecture, we will learn how to get the error on *m* and *b* due to random errors y (see a text book for errors in x). Next week, we'll learn how to get the systematic errors.

Slope and Intercept

Instead of using the solver, let's calculate the slope and intercept analytically (let's ignore errors on x to simplify for now. See a standard text for full derivation).

$$\chi^2 = \sum_i \frac{(y_i - b - mx_i)^2}{\sigma_i^2}$$

Want to find the values of *m* and *b* that gives the min value for chi^2. You remember how to find the maximums and minimums of a function using calculus, right?

$$\frac{\partial \chi^2}{\partial b} = -2\sum_i \frac{1}{\sigma_i^2}(y_i - b - mx_i) = 0$$

$$\frac{\partial \chi^2}{\partial m} = -2\sum_i \frac{x_i}{\sigma_i^2}(y_i - b - mx_i) = 0$$

Minimizing χ^2, cont'd.

Rearrange a little bit:

$$\left(\sum_i \frac{y_i}{\sigma_i^2}\right) = b\left(\sum_i \frac{1}{\sigma_i^2}\right) + m\left(\sum_i \frac{x_i}{\sigma_i^2}\right)$$

$$\left(\sum_i \frac{x_i y_i}{\sigma_i^2}\right) = b\left(\sum_i \frac{x_i}{\sigma_i^2}\right) + m\left(\sum_i \frac{x_i^2}{\sigma_i^2}\right)$$

Two equations and two unknowns (*m* and *b*). Solve for *m* and *b*.

$$b = \frac{1}{\Delta}\left[\left(\sum_i \frac{x_i^2}{\sigma_i^2}\right)\left(\sum_i \frac{y_i}{\sigma_i^2}\right) - \left(\sum_i \frac{x_i}{\sigma_i^2}\right)\left(\sum_i \frac{x_i y_i}{\sigma_i^2}\right)\right]$$

$$m = \frac{1}{\Delta}\left[\left(\sum_i \frac{1}{\sigma_i^2}\right)\left(\sum_i \frac{x_i y_i}{\sigma_i^2}\right) - \left(\sum_i \frac{x_i}{\sigma_i^2}\right)\left(\sum_i \frac{y_i}{\sigma_i^2}\right)\right]$$

$$\Delta = \det\begin{bmatrix} \left(\sum_i \frac{1}{\sigma_i^2}\right) & \left(\sum_i \frac{x_i}{\sigma_i^2}\right) \\ \left(\sum_i \frac{x_i}{\sigma_i^2}\right) & \left(\sum_i \frac{x_i^2}{\sigma_i^2}\right) \end{bmatrix}$$

Error on Slope and Intercept

Our calculated value for *m* and *b* depend on the measured values x_i, y_i. We can therefore get the errors on *m* and *b* by using the standard propagation of errors formula on our formulas for these quantities.

Let's do this for the intercept. The derivation for the error on the slope is similar.

Let's also just do this for the simple case when the errors on the y's are all the same ($sigma_i$ -> sigma)

Error on Intercept

$$\Delta = \det\begin{bmatrix} \left(\sum_i \frac{1}{\sigma_i^2}\right) & \left(\sum_i \frac{x_i}{\sigma_i^2}\right) \\ \left(\sum_i \frac{x_i}{\sigma_i^2}\right) & \left(\sum_i \frac{x_i^2}{\sigma_i^2}\right) \end{bmatrix}$$

When sigma_i -> sigma

$$\Delta = \frac{1}{\sigma^4}(N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2$$

this is just a constant (that does not depend on y_i.)

Error on Intercept

$$b = \frac{1}{\Delta}\left[\left(\sum_i \frac{x_i^2}{\sigma_i^2}\right)\left(\sum_i \frac{y_i}{\sigma_i^2}\right) - \left(\sum_i \frac{x_i}{\sigma_i^2}\right)\left(\sum_i \frac{x_i y_i}{\sigma_i^2}\right)\right]$$

If the sigma_i's are all the same (sigma), this becomes

$$b = \frac{1}{\Delta\sigma^4}\left[\left(\sum_i x_i^2\right)\left(\sum_i y_i\right) - \left(\sum_i x_i\right)\left(\sum_i x_i y_i\right)\right]$$

Error on intercept

$$b = \frac{1}{\Delta\sigma^4}\left[\left(\sum_i x_i^2\right)\left(\sum_i y_i\right) - \left(\sum_i x_i\right)\left(\sum_i x_i y_i\right)\right]$$

$$\frac{\partial b}{\partial y_i} = \frac{1}{\Delta\sigma^4}\left[\left(\sum_j x_j^2\right) - x_i\left(\sum_j x_j\right)\right]$$

$$\sigma_b = \frac{1}{\Delta\sigma^4}\sqrt{\sum_i(\sum_j x_j^2 - x_i \sum_j x_j)^2\sigma^2}$$

$$= \frac{1}{\Delta\sigma^3}\sqrt{\sum_i(\sum_j x_j^2)^2 - 2x_i(\sum_j x_j)(\sum_j x_j^2) + x_i^2(\sum_j x_j)^2}$$

$$= \frac{1}{\Delta\sigma^3}\sqrt{N(\sum_j x_j^2)^2 - 2(\sum_j x_j)^2(\sum_j x_j^2) + (\sum_j x_j^2)(\sum_j x_j)^2}$$

$$= \frac{1}{\Delta\sigma^3}\sqrt{N(\sum_j x_j^2)^2 - (\sum_j x_j^2)(\sum_j x_j)^2}$$

$$= \frac{1}{\Delta\sigma^3}\sqrt{(N(\sum_j x_j^2) - (\sum_j x_j)^2)\sum x_j^2}$$

$$= \frac{1}{\Delta\sigma}\sqrt{\frac{(N(\sum_j x_j^2) - (\sum_j x_j)^2)}{\sigma^4}\sum x_j^2}$$

$$= \frac{1}{\sigma}\sqrt{\frac{\sum x_j^2}{\Delta}}$$

Error on Intercept

$$\sigma_b = \frac{1}{\sigma}\sqrt{\frac{\sum x_j^2}{\Delta}} = \frac{1}{\sigma}\sqrt{\frac{\sum x_j^2}{\left(N\sum x_j^2 - (\sum x_j)^2\right)/\sigma^4}} = \sigma\sqrt{\frac{\sum x_j^2}{\left(N\sum x_j^2 - (\sum x_j)^2\right)}}$$

Look in your linear fitter spread sheet and you'll see this linked to the error on intercept sheet

Oscilloscope

Oscilloscope review

Next week

See you next week!