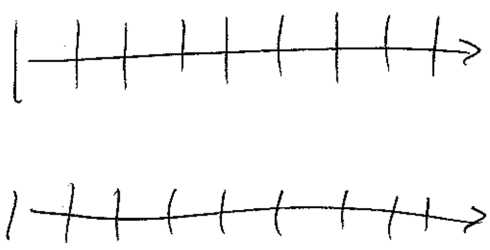
Two Beam Interference



$$E_1 = Ae^{i(kx_1-\omega t)}$$

$$E_2 = Ae^{i(kx_2-\omega t)}$$

• P

Beams overlap here.

$$E_p = E_1+E_2 = 2A\underbrace{e^{i(k\bar{x}-\omega t)}}_{\text{wave factor}}\underbrace{\cos\left(\frac{k\delta}{2}\right)}_{\text{amplitude factor}}$$

$$\delta \equiv x_2-x_1$$

$$\bar{x} \equiv \frac{x_2+x_1}{2}$$

Usually we don't care about the wave factor (oscillating too fast to see).

If we only want the amplitude factor, then

$$\text{Intensity} \propto E_pE_p^* = 4A^2\cos^2\frac{k\delta}{2}$$

$$\left(I = \frac{\varepsilon_0 c}{2} E_pE_p^* = \varepsilon_0 c\, 2A^2\cos^2\frac{k\delta}{2} \right)$$

More generally, $E_pE_p^* = 4A^2\cos^2\frac{\Delta\phi}{2}$, $\Delta\phi$ = phase difference

For a single beam, the result would be

$$E_pE_p^* = A^2$$

For interfering beams of equal intensity, the intensity maximum is 4 times what it would be for one beam.

To observe interference we need:

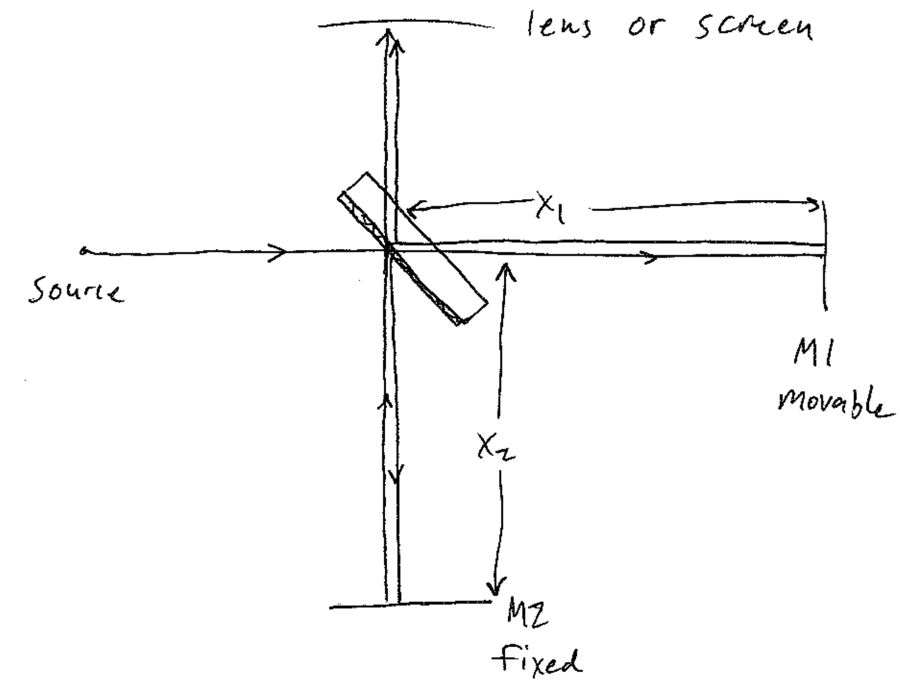
- same polarization (usually get this automatically by using the same source to create the two beams.)

- coherent waves

  – real light waves are not perfect cos functions. Coherence time is amount of time light wave remains predictable. Coherence length is distance light wave remains predictable.

He-Ne lasers have coherence lengths of $\sim$(20 cm – 10 m)
For white light, coherence length is about one wavelength ($\frac{1}{2}\mu m$)

Michelson Interferometer



$$I \propto 4A^2 \cos^2 \frac{\Delta\phi}{2}$$

path difference $= 2(x_1 - x_2) = \delta$

- phase due to path difference $= k\delta = 2k(x_1 - x_2)$
- phase due to internal reflection of beam along M1 $= \pm\pi$

total phase difference $= \dfrac{2k(x_2 - x_1) \pm \pi}{2} = m\pi$ for light fringes

$m = 0, 1, 2, 3$

$$k = \frac{2\pi}{\lambda}$$

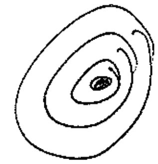
"fringe order"

$$\boxed{\frac{2(x_2 - x_1)}{\lambda} = m \pm \frac{1}{2}}, \quad m = 0, 1, 2, 3$$

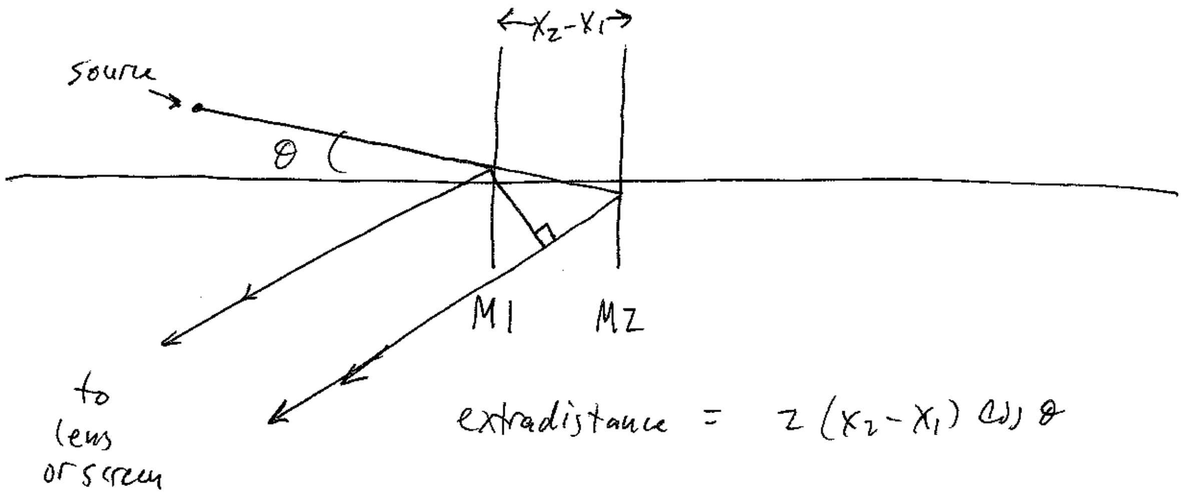
bright spot – constructive interference.

$$\boxed{\frac{2(x_2-x_1)}{\lambda} = m, \quad m = 0, 1, 2, 3 \ldots}$$ destructive interference.

In practice you don't see one spot, you see rings:



This is because off-axis points have additional phase differences due to the path length difference:



extradistance $= 2(x_2 - x_1)\cos\theta$

If $x_2 \equiv x_1$, then only one fringe will be seen (no off-axis phase difference)

~~If $x_2 = x_1 + 100\lambda$~~

otherwise many rings will be seen.

In some cases ~~Typically~~, we don't care about which order of fringe we are observing. Instead, we move M2 and count the number of fringes that pass by. Thus

$$\boxed{\Delta m = \frac{2(x_2-x_1)}{\lambda}}$$

Can use this to measure the wavelength of the light.

Sodium Lamp

Two closely spaced wavelengths: λ, λ'

Each makes its own fringe pattern.

Suppose the Michelson interferometer gives a maximum for both wavelengths:

λ $\qquad$ λ'

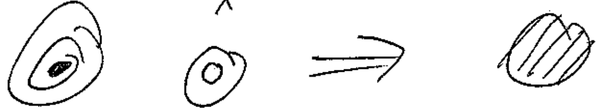
Fringe order m $\qquad$ Fringe order m'

$m = m' + N$

Then $\quad \dfrac{2d_1}{\lambda} = \dfrac{2d_1}{\lambda'} + N \,, \quad d_1 \equiv x_2 - x_1$

Now increase x_2 (move mirror). After a while, the fringe pattern disappears:

λ $\qquad$ λ'



Then keep moving, and fringes reappear:

Now $\quad x_2 - x_1 \equiv d_2$

$$\frac{2d_2}{\lambda} = \frac{2d_2}{\lambda'} + \underbrace{N+1}$$

m and m' differ by one more unit now

$$\frac{2(d_2-d_1)}{\lambda} = \frac{2(d_2-d_1)}{\lambda'} + 1$$

$$\frac{2\Delta d}{\lambda} - \frac{2\Delta d}{\lambda'} = 1$$

$$\lambda - \lambda' = \frac{\lambda\lambda'}{2\Delta d} \implies \boxed{\Delta\lambda = \frac{\lambda^2}{2\Delta d}} \quad \lambda \sim \lambda'$$

Δd is change in mirror distance between two constructive interference

Rewrite

Lecture 9 - Single Slit Diffraction 10/31/06 (1)

Review from last week

$I \propto 4A^2 \cos^2 \frac{\Delta\phi}{2}$ Two Beam interference

$\Delta\phi$ = all phase difference between two beams.

Michelson interferometer: total phase difference $= 2k(x_1 - x_2) + \pi$.

Constructive interference: $\cancel{2k(x_1-x_2)}$ $\frac{2(x_1 - x_2)}{\lambda} = m + \frac{1}{2}$, $m = 0, 1, 2$.

destructive interference: $\frac{2(x_1 - x_2)}{\lambda} = m$, $m = 0, 1, 2 \ldots$

Measuring the wavelength: keep M1 fixed, move M2; count fringes:

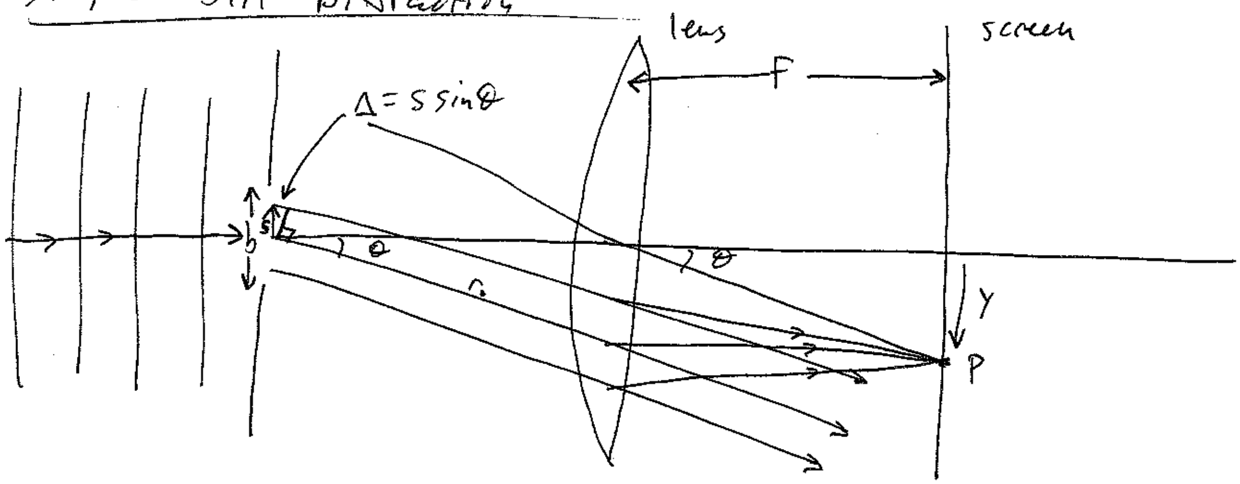
$$\Delta m = \frac{2\Delta x_2}{\lambda}$$

Sodium Lamp: Two closely spaced wavelengths λ and λ'

$$\Delta\lambda = \lambda - \lambda' \approx \frac{\lambda^2}{2\Delta d}$$

$\uparrow$ the distance M2 moves between to case of maximum fringe visibility.

Single Slit Diffraction



Fraunhofer Diffraction (Far Field Diffraction): Observe interference pattern at infinity as a function of θ. $L \gg b^2/\lambda$ $\leftarrow$ Far Field Condition.

Use a lens to move diffraction pattern from infinity to a screen

Each point in the aperture is a source of spherical waves.

Field at P due to a single point on the aperture:

$$dE_p = \underbrace{\left(\frac{E_L\,ds}{r}\right)}_{\substack{\text{Amplitude factor} \\ \text{for spherical} \\ \text{waves}}} \underbrace{e^{i(kr-\omega t)}}_{\substack{\text{phase} \\ \text{factor}}}$$

$$\left(I \propto \frac{1}{r^2},\ \text{so } E \propto \frac{1}{r}\right)$$

Let r_0 be the distance travelled by the wavelet originating at the center of the aperture.

Then $\Delta = s\sin\theta$

$$dE_p = \left(\frac{E_L\,ds}{r_0+\Delta}\right) e^{i(k(r_0+\Delta)-\omega t)} \approx \frac{E_L\,ds}{r_0}\, e^{i(kr_0-\omega t)}\, e^{ik\Delta}$$

can ignore Δ compared to r_0

but can't ignore phase due to Δ

Integrate over the aperture to get the total field at Point P: $\Delta = s\sin\theta$

$$E_p = \int dE_p = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \int_{-\frac{b}{2}}^{b/2} e^{iks\sin\theta}\, ds$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left(\frac{e^{iks\sin\theta}}{ik\sin\theta}\right)_{-b/2}^{b/2}$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left(\frac{e^{ikb\sin\theta/2} - e^{-ikb\sin\theta/2}}{ik\sin\theta}\right)$$

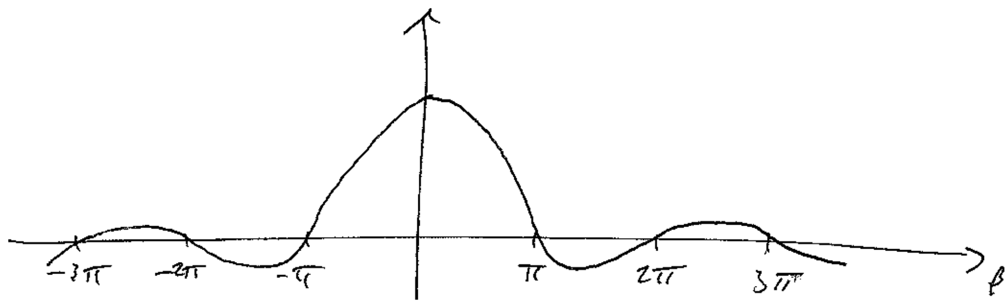
$$\beta \equiv \tfrac{1}{2}kb\sin\theta$$

$$E_p = \frac{E_L b}{r_0} e^{i(kr_0-\omega t)} \frac{2i\sin\beta}{2i\beta} = \boxed{\frac{E_L b}{r_0} e^{i(kr_0-\omega t)} \left(\frac{\sin\beta}{\beta}\right)}$$

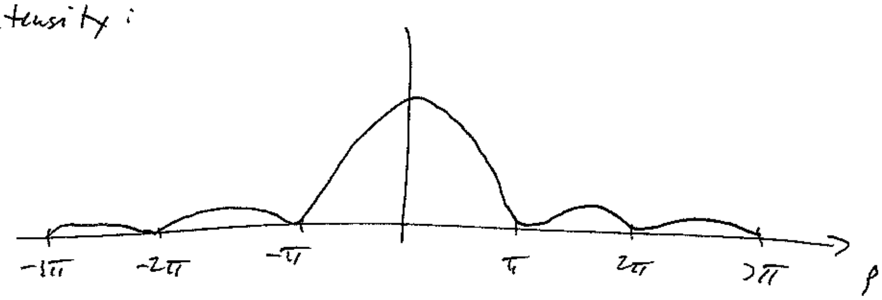
$$\boxed{I = \frac{\varepsilon_0 c}{2} E_p E_p^* = \frac{\varepsilon_0 c}{2}\left(\frac{E_L b}{r_0}\right)^2 \frac{\sin^2\beta}{\beta^2}}, \quad \beta = \tfrac{1}{2}kb\sin\theta$$

$$I = I_0 \frac{\sin^2\beta}{\beta^2}$$

Electric Field amplitude:



Intensity:



Zeros occur when $\beta = \pm m\pi$, $m = \pm 1, 2, 3,$

but not $m = \emptyset$!

$$\tfrac{1}{2} k b \sin\theta = m\pi$$

$$\frac{b}{\lambda} \sin\theta = m$$

$$\boxed{m\lambda = b\sin\theta} \text{ zeros}$$

When using a lens, we can write θ in terms of the position on the screen:

$$\frac{y}{f} = \tan\theta \approx \sin\theta$$

$$\sin\theta \approx \frac{y}{f}$$

$$m\lambda = \frac{by}{f}$$

$$\boxed{y_m = \frac{m\lambda f}{b}} \text{ zeros with a lens.}$$

width of the central maximum:

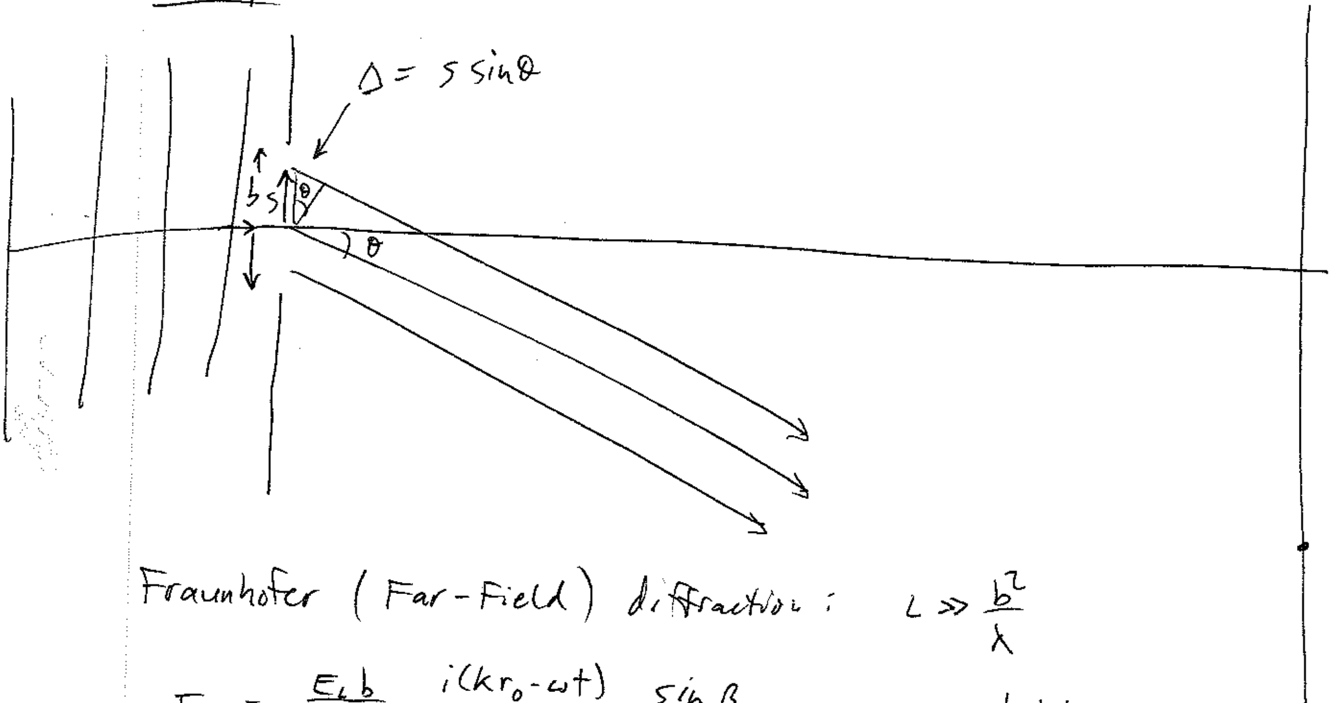
$$\Delta\theta = \frac{2\lambda}{b}$$

inversely proportional to the aperture width.

Rewrite

Lecture 10 - Double and Multiple Slit Diffraction 11/7/06

Recap



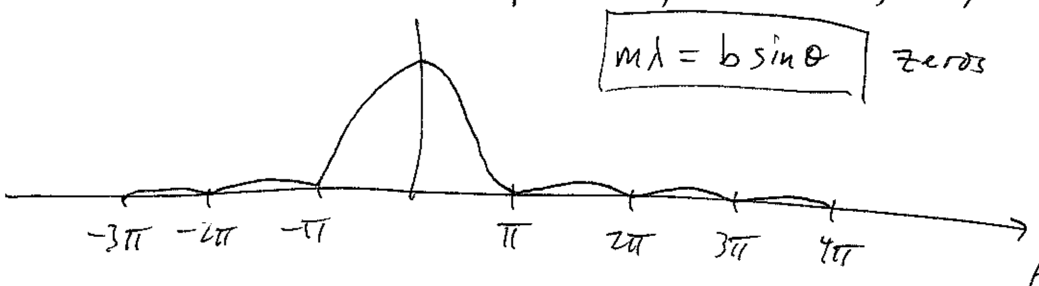
Fraunhofer (Far-Field) diffraction: $L \gg \frac{b^2}{\lambda}$

$$E_p = \frac{E_L b}{r_0} e^{i(kr_0 - \omega t)} \frac{\sin\beta}{\beta}, \quad \beta \equiv \tfrac{1}{2} kb\sin\theta$$

$$I = \frac{\varepsilon_0 c}{2} E E_p^* = \frac{\varepsilon_0 c}{2}\left(\frac{E_L b}{r_0}\right)^2 \frac{\sin^2\beta}{\beta^2} = I_0 \frac{\sin^2\beta}{\beta^2}$$

zeros at $\beta = m\pi$, $m = \pm 1, \pm 2, \ldots$, but not $m = \emptyset$!

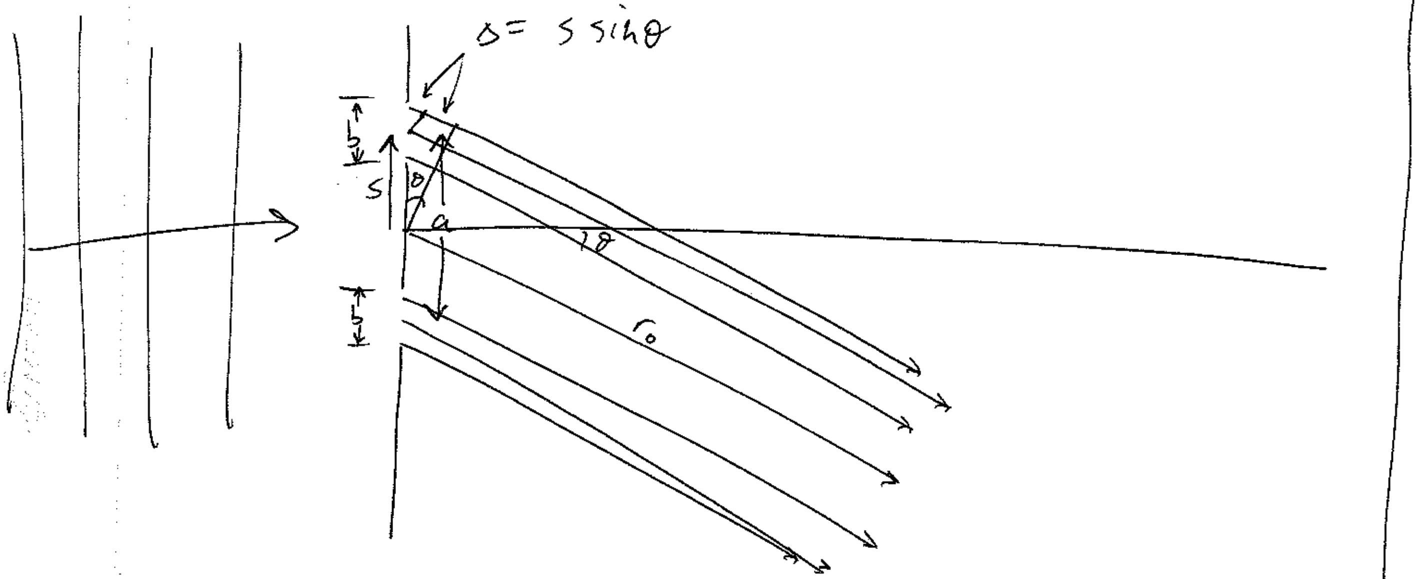
$\boxed{m\lambda = b\sin\theta}$ zeros



If we use a lens, and put the screen in the focal plane, then the zeros are located at

$\boxed{y_m = \frac{m\lambda F}{b}}$ zeros, F = focal length
y_m = position on the screen

Double Slit Diffraction



$$dE_p = \frac{E_L ds}{r} e^{i(kr-\omega t)} \qquad r = r_0 + \Delta = r_0 + s\sin\theta$$

$$E_p = \int dE_p = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \int_{\text{slits}} e^{ik\Delta} ds$$

$$= \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left[\int_{-\frac{1}{2}(a+b)}^{-\frac{1}{2}(a-b)} e^{iks\sin\theta} ds + \int_{+\frac{1}{2}(a-b)}^{\frac{1}{2}(a+b)} e^{iks\sin\theta} ds \right]$$

$$= \frac{E_L}{r_0} \frac{e^{i(kr_0-\omega t)}}{ik\sin\theta} \left[e^{-\frac{1}{2}k\sin\theta(a-b)} - e^{-\frac{1}{2}k\sin\theta(a+b)} + e^{\frac{1}{2}k\sin\theta(a+b)} - e^{\frac{1}{2}(a-b)k\sin\theta} \right]$$

$\beta = \frac{1}{2} kb\sin\theta$

$\alpha = \frac{1}{2} ka\sin\theta$

$$= \frac{E_L}{r_0} \frac{e^{i(kr_0-\omega t)}}{ik\sin\theta} \left[e^{-i\alpha} e^{i\beta} - e^{-i\alpha} e^{-i\beta} + e^{i\alpha} e^{i\beta} + e^{-i\alpha} e^{-i\beta} \right]$$

$$= \frac{E_L(\frac{1}{2}b)}{r_0} \frac{e^{i(kr_0-\omega t)}}{\underbrace{(\tfrac{1}{2}b)ik\sin\theta}_{\beta}} \left[\underbrace{\left(e^{-i\alpha} + e^{i\alpha}\right)}_{2\cos\alpha} \underbrace{\left(e^{i\beta} - e^{-i\beta}\right)}_{2i\sin\beta} \right]$$

$$= \frac{2E_L b}{r_0} e^{i(kr_0-\omega t)} \frac{\sin\beta}{\beta} \cos\alpha$$

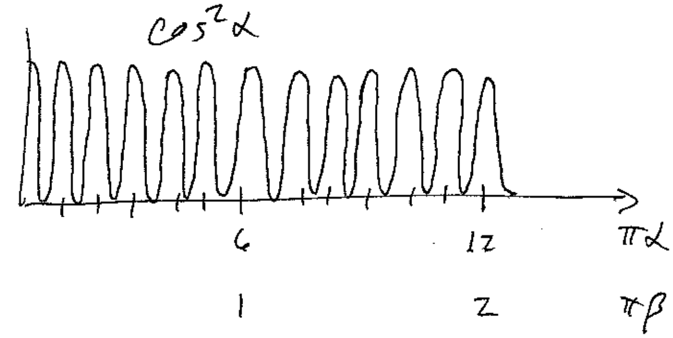
$$I = \frac{\varepsilon_0 c}{2} \frac{4 E_1^2 b^2}{r_0^2} \frac{\sin^2\beta}{\beta^2} \cos^2\alpha$$

$$= 4 I_{0ss} \frac{\sin^2\beta}{\beta^2} \cos^2\alpha \qquad \alpha = \tfrac{1}{2} ka \sin\theta = \frac{\pi a \sin\theta}{\lambda}$$

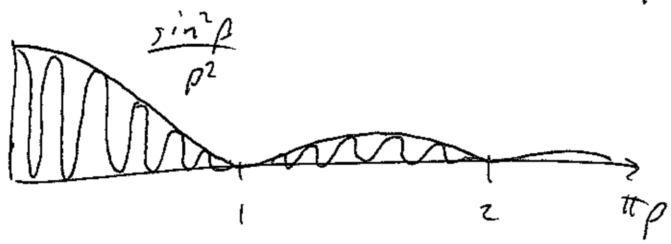
single slit intensity

$$\beta = \tfrac{1}{2} bk \sin\theta = \frac{\pi b \sin\theta}{\lambda}$$

Suppose $a = 6b$. Then $\alpha = 6\beta$



"Interference term"



"Diffraction term"

When slit spacing is an integral number of slit widths, we always get missing maxima.

Diffraction minima: $m\lambda = b\sin\theta$, $m = \pm 1, \pm 2,$

Interference maxima: $n\lambda = a\sin\theta$, $n = 0, \pm 1, \pm 2, \ldots$

Number of peaks in central diffraction fringe $= 2\left(\frac{a}{b}\right) - 1$.

N slits:

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

Interference Factor: $\frac{\sin N\alpha}{\sin\alpha}$ indeterminate whenever $\alpha = 0, \pm\pi, \ldots$

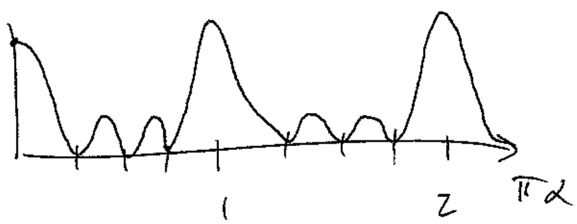
$$\lim_{\alpha \to m\pi} \frac{\sin N\alpha}{\sin\alpha} = \lim_{\alpha \to m\pi} \frac{N\cos N\alpha}{\cos\alpha} = N$$

Therefore interference maxima are N^2 more intense than they would have been for 1 slit.

$$I = N^2 I_{0ss} \frac{\sin^2 \beta}{\beta^2} \quad \text{when} \quad \alpha = n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

$\frac{\sin^2 N\alpha}{\sin^2 \alpha}$: Suppose $N = 4$

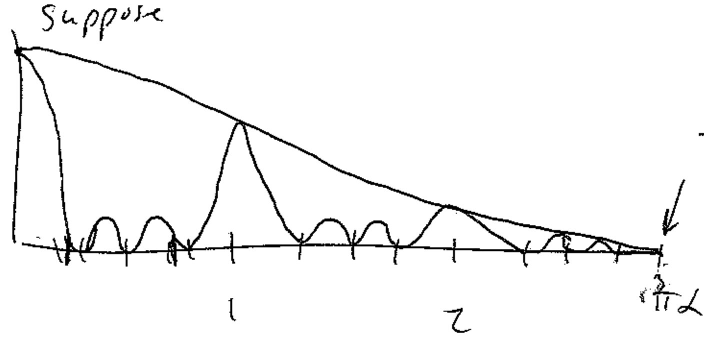
Interference Term.



Zeros when $\alpha = \frac{n\pi}{N}$

$N-1$ zeros between principle maxima, and $N-2$ secondary maxima between principle maxima.

Suppose



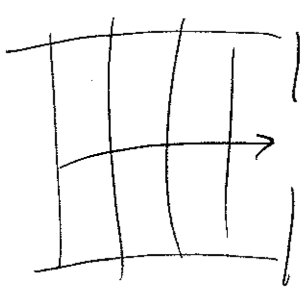
third principle maxima is missing.

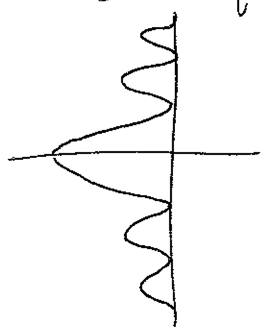
Therefore $\beta = \frac{\alpha}{3}$

$b = \frac{a}{3}$

Babinet's Principle

Complementary obstruction produce the same diffraction pattern, except for the beam spot:





Use this to measure the width of an obstruction, like a human hair.