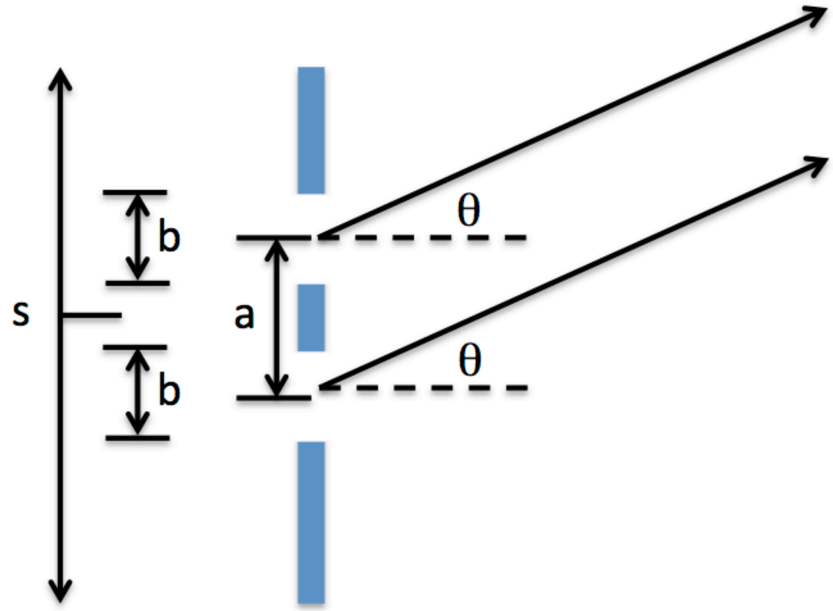
Homework #11 - Phys 273

1) Consider a two-slit diffraction experiment with the following geometry:



The electric field amplitude on a distant screen (E_p) at angle (θ) is determined by integrating over the slits:

$$E_P = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \left[\int_{-(a+b)/2}^{-(a-b)/2} e^{iks\sin\theta} ds + \int_{(a-b)/2}^{(a+b)/2} e^{iks\sin\theta} ds \right]$$

where (a) is the distance between the slits and (b) is the slit width. Evaluate the integrals and show that

$$E_P = \frac{2E_L b}{r_0} e^{i(kr_0 - \omega t)} \frac{\sin\beta}{\beta} \cos\alpha$$

where $\alpha = \frac{1}{2} ka\sin\theta, \beta = \frac{1}{2} kb\sin\theta$.

2) Sketch the diffraction patterns that you would observe on a screen 1 m from an aperture illuminated by a He-Ne laser (wavelength = 633 nm), for the following cases:

a) a single slit, with slit width = 20 microns.
b) two slits, slit width = 20 microns, slit spacing = 80 microns.

Draw these diffraction patterns as a function of position on the screen, with the x-axis in units of meters, and label the position of the first zero due to the "slit width" term.

3) In the second half of the nineteenth century it was believed that the speed of propagation of light waves depends on the frame of reference in which the waves are observed. In 1887, Michelson and Morley performed a famous experiment at the Case Institute (now Case Western Reserve University) in Cleveland, Ohio. Using a Michelson interferometer, they hoped to detect the motion of the earth through space by observing the small change in the velocity of light that would occur due to the earth's motion.

Imagine that a Michelson interferometer is oriented so that one arm points in the direction of earth's motion around the sun, and that the other arm is transverse to the earth's motion. The interferometer is illuminated with a He-Ne laser, with wavelength 633 nm, and the length of each arm is ten meters. Six hours later, both arms are perpendicular to the earth's motion, because the earth has rotated.

If we assume that the speed of light for motion parallel to the earth's velocity is $(c + v_e)$, and the speed of light perpendicular to the earth's motion is simply (c), how many fringes should be observed to pass by a detector during the six hours that it takes the earth to rotate from the first configuration to the second? (c) is the speed of light in vacuum (3.0×10^8 m/s), and (v_e) is the Earth's orbital velocity around the sun, which is 3.0×10^4 m/s.

As it turns out, no fringe shift can be seen in this experiment because the velocity of light is independent of the frame of reference. This is a basic consequence of special relativity, and today the Michelson-Morley experiment is considered to be the first experimental evidence for the theory of relativity.