Why you should hate sine and cosine.

Sine and Cosine are not a good way of representing an oscillator or a wave.

Reason #1 They don't obey the normal rules of algebra.

Ex: $\frac{\cos\theta}{s} \stackrel{?}{=} \text{co}\theta$?!? [of course not]

s does not divide out!

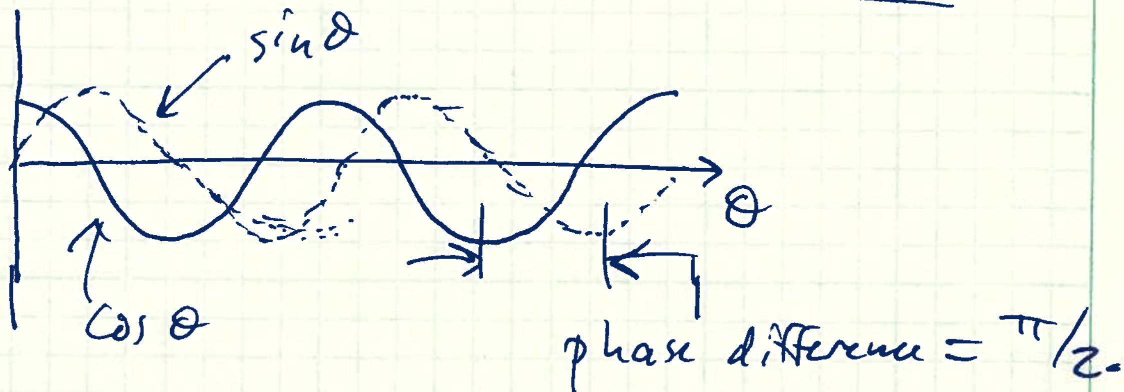
Since they are not normal algebraic operations, we have to ~~use~~ refer to long tables of trigonometric identities to use them. Trig identities are excellent opportunities to make mistakes. The table of trig identities in Hirose & Lonngren contains several errors! (Appendix B) (of Intro to Wave Phenomena)

Reason #2 Sine & Cosine obscure the true Amplitude and Phase, and Frequency of an oscillation.

Ex#1 $f(\theta) = \cos\theta$, $g(\theta) = \sin\theta$ } what is the phase difference between $f(\theta)$ & $g(\theta)$?

It appears to be zero, because the argument for both is just θ. But sine & cosine have a built-in phase difference of $\underline{\pi/2}$:

~~so the~~



Ex #2 What's the phase difference between

$$f(\theta) = \cos(\theta - \pi/2)$$
$$g(\theta) = \sin(\theta)$$
?

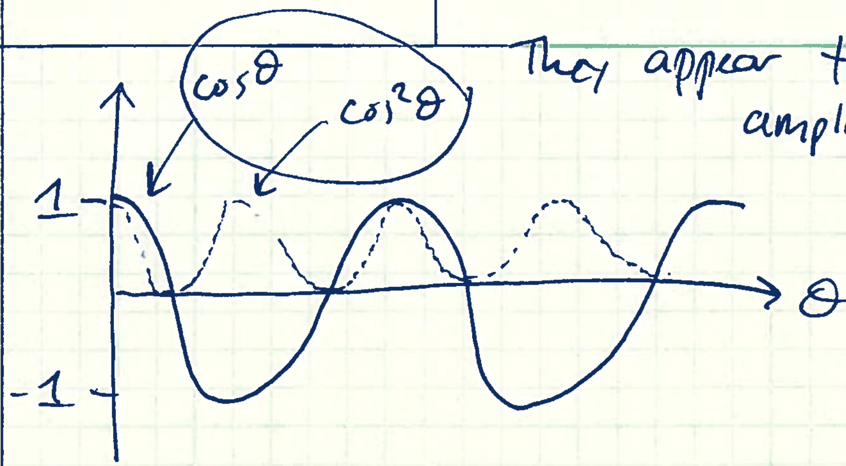
Answer: It appears to be $-\pi/2$, but it's really zero.

Ex #3 What's the amplitude of $A\cos(\omega t + \delta)$?

Answer: It depends what you mean by amplitude. The full peak-to-peak variation ~~of~~ is $\underline{2A}$. In this sense, A is really the half-amplitude.

Ex #4 What's the ~~[crossed out]~~ amplitude of ~~[crossed out]~~ $[A\cos(\omega t + \delta)]^2$?

Answer: ~~by~~ after squaring the function, now the full peak-to-peak amplitude is simply A^2, not $(2A)^2$.



They appear to have the same amplitude, but $\cos\theta$ has a full ~~variation~~ peak-to-peak variation twice as large as $\cos^2\theta$.

EX #5 Does $\cos(\omega t)$ have the same frequency as $\cos^2(\omega t)$?

Answer: No $\rightarrow$ $\cos^2(\omega t)$ has twice the frequency as $\cos(\omega t)$.

$$\cos^2\omega t = \frac{1}{2}\left[1+\cos(2\omega t)\right]$$ ← we have to use a trig identity to see that $\cos^2(\omega t)$ has twice the frequency.

~~Ex #6 Does $\cos^4(\omega t)$ have the same~~

$\Rightarrow$ An oscillation is nothing but an amplitude, a phase, and a frequency, and sine & cosine are misleading about all three.

Reason #3 (why you should hate sine & cosine)

You might try to simplify your life by only using sine or only using cosine, but as soon as you differentiate, sine turns into cosine, and vice-versa. So you are (more or less) forced to use both. Since the equations of motion of physics are ODEs, we can't avoid taking derivatives.

The fundamental problem with sine & cosine are that they are designed to work well in geometry. But there is nothing particularly geometrical about a mass bouncing up and down on a spring. Sine and Cosine correctly describe a mass on a spring, but they are not the most natural way or simplest way to describe it. They are intended for geometry, not dynamics.

Simple Harmonic Oscillator again.

Equation of Motion is $\boxed{\ddot{x} + \omega_0^2 x = 0}$

This equation demands a function $x(t)$ which is proportional to its own second derivative, (with a minus sign) Sine and Cosine happen to satisfy this, but so do exponential functions:

$$x(t) = e^{\omega_0 t} \leftarrow \text{guessed solution}$$

$$\ddot{x}(t) = \omega_0^2 e^{\omega_0 t} = \omega_0^2 x(t)$$

Try it: $\ddot{x} + \omega_0^2 x =$

$$= \omega_0^2 e^{\omega_0 t} + \omega_0^2 e^{\omega_0 t} = 2\omega_0^2 e^{\omega_0 t} \quad ?!?$$

$$\neq \emptyset \;!!$$

We wanted a (−) sign here, but we didn't get it. We need to replace (ω_0) with $(i\omega_0)$ where $i = \sqrt{-1}$.

$$x(t) = e^{i\omega_0 t} \longleftarrow \text{better guess}$$

$$\ddot{x}(t) = -\omega_0^2 e^{i\omega_0 t}$$

$$\ddot{x} + \omega_0^2 x$$

$$= -\omega_0^2 e^{i\omega_0 t} + \omega_0^2 e^{i\omega_0 t} = \emptyset \;\checkmark\; \text{Yes.}$$

So $e^{i\omega_0 t}$ is a solution to the simple harmonic oscillator equation of motion. It's a much ~~better~~ simpler way to represent the solution than sine & cosine because ~~it obeys all the normal rules of algebra $\Rightarrow$ (no more trig identities!)~~, ~~and because~~

1. it obeys the normal rules of algebra (no more trig identities!)
2. It makes the amplitude, phase, and frequency much more obvious, and
3. taking derivatives is easy. (no more switching between sine & cosine.)

But it does raise some questions:

a) What does it mean to exponentiate the square root of -1?

b) How can a complex function (real and imaginary) represent the position of a mass on a spring, which must always be a <u>real number</u>?