Review

Mechanical impedance of a string:

$$Z = \frac{\text{transverse force}}{\text{transverse velocity}}$$

$$Z = \frac{T}{v_p} \quad \begin{matrix}\leftarrow \text{Tension} \\ \leftarrow \text{phase velocity}\end{matrix}$$

$$= \rho v_p$$

mass density ↗ ↖ phase velocity

Reflection & Transmission at an impedance boundary

$$\frac{\text{transmitted amplitude}}{\text{incoming amplitude}} = \frac{A_2}{A_1} = \frac{2Z_1}{Z_1 + Z_2}$$

$$\frac{\text{reflected amplitude}}{\text{incoming amplitude}} = \frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

Energy Transport by a wave

$$\text{Power} = \text{Energy Transmission} = \frac{1}{2} Z \omega^2 A^2$$

impedance of string ↗ ↑ frequency ↖ Amplitude of wave

Transmission Lines

L_0 = Inductance per unit length

C_0 = Capacitance per unit length

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

Z_0 = "characteristic impedance" $\equiv \frac{V_0}{I_0}$

$$Z_0 = \sqrt{\frac{L_0}{C_0}}$$ for a transmission line.

Voltage ~~Ref~~ Reflection & Transmission

$$\frac{\text{Reflected Amp.}}{\text{Incoming Amp.}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_-}{V_+}$$

$$\frac{\text{Transmitted Amp.}}{\text{Incoming Amp.}} = \frac{V_L}{V_+} = \frac{2Z_L}{Z_L + Z_0}$$

Current Reflection & Transmission

$$\frac{\text{Reflected Amp}}{\text{Incoming Amp}} = \frac{I_-}{I_+} = \frac{Z_0 - Z_L}{Z_0 + Z_L}$$

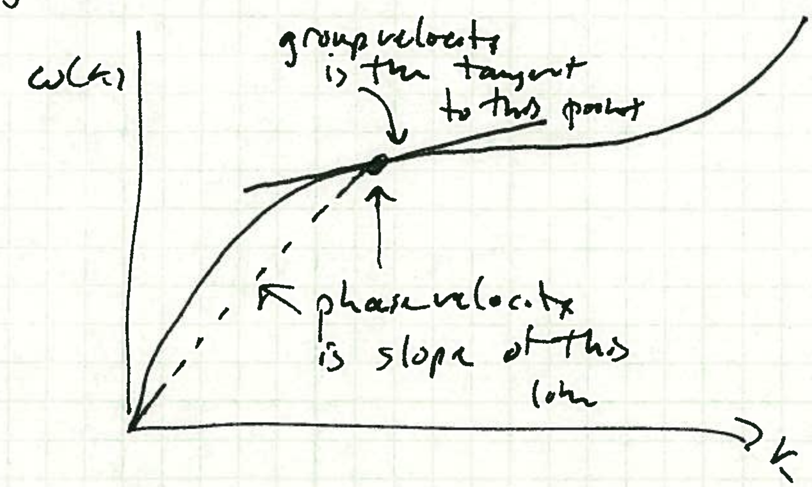
$$\frac{\text{Transmitted Amp}}{\text{Incoming Amp}} = \frac{I_L}{I_+} = \frac{2Z_0}{Z_L + Z_0}$$

Dispersion

$$\text{phase velocity} \equiv \frac{\omega(k)}{k} \quad \text{by definition}$$

$$\text{group velocity} \equiv \frac{\partial \omega}{\partial k}$$

If the phase velocity depends on k, then pulses will disperse. Information travels at the speed of the pulse envelope, which is the group velocity:



A pulse can be transmitted through a dispersive medium by multiplying it by a carrier wave:

$$z(x) = (\text{pulse}) \times (\text{carrier}) = f(x)\, e^{ik_0 x}$$

$f(x)$: pulse

$e^{ik_0 x}$: carrier wave, short wavelength

As long as the dispersion relation is approximately linear over the range of wave numbers included in the pulse, then

$$z(x,t) = \underbrace{F(x - v_g t)}_{\text{pulse envelope}} \, \underbrace{e^{i(k_c x - \omega_c t)}}_{\text{carrier wave}}$$

pulse envelope propagates without changing shape

carrier wave

where v_g = group velocity = $\frac{\partial \omega}{\partial k}(k = k_c)$