1) a) $\psi_{signal}(x) = \begin{cases} C e^{ik_0 x} & , -\frac{L}{2} < x < \frac{L}{2} \\ \emptyset & , \text{otherwise} \end{cases}$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \, \psi_{signal}(x) e^{-ikx}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\frac{L}{2}}^{\frac{L}{2}} C\left(e^{ik_0 x}\right) e^{-ikx} \, dx$$

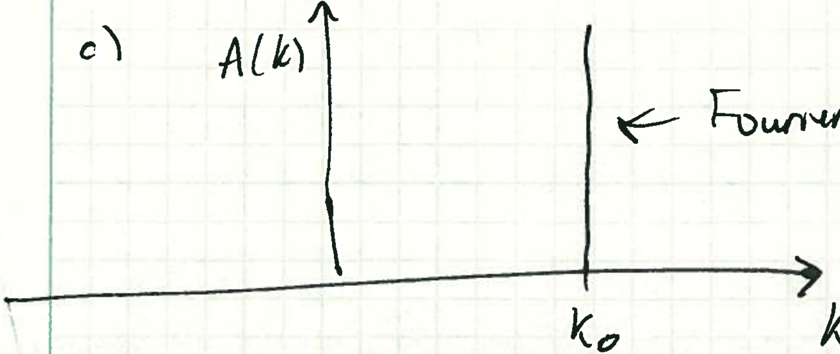
$$= \frac{C}{\sqrt{2\pi}} \left(\left. \frac{e^{i(k_0-k)x}}{i(k_0-k)} \right|_{-\frac{L}{2}}^{\frac{L}{2}} \right)$$

$$= C\sqrt{\frac{2}{\pi}} \frac{1}{(k_0-k)} \underbrace{\left(\frac{e^{i(k_0-k)\frac{L}{2}} - e^{-i(k_0-k)\frac{L}{2}}}{2i} \right)}_{\sin\left((k_0-k)\frac{L}{2}\right)}$$

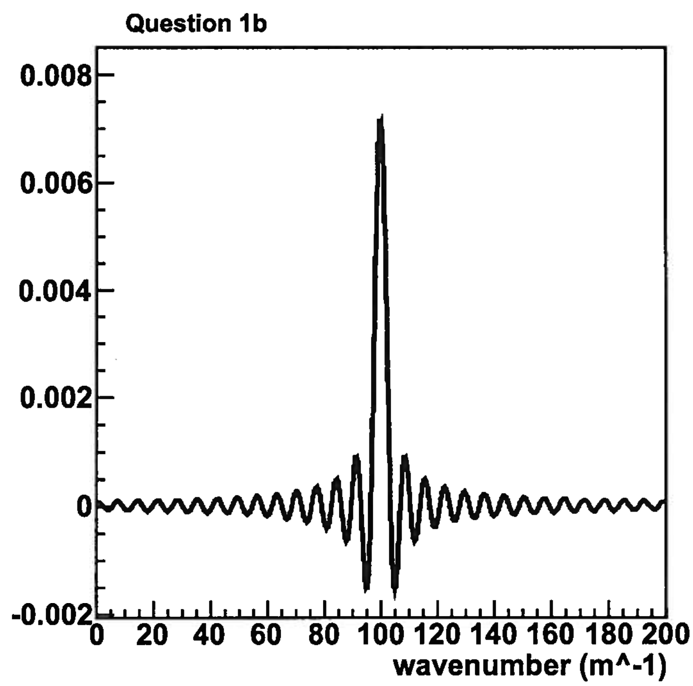
$$= C\sqrt{\frac{2}{\pi}} \left(\frac{\sin\left((k_0-k)\frac{L}{2}\right)}{(k_0-k)} \right)$$

b) see attached

c)



← Fourier transform of a perfect harmonic wave.



Question 1b
0.008
0.006
0.004
0.002
0
-0.002
0
20
40
60
80
100
120
140
160
180
200
wavenumber (m^-1)

2) a) $V_{phase} = \dfrac{\omega(k)}{k}$

$$= \frac{\sqrt{\omega_e^2 + (ck)^2}}{k}$$

$$= \sqrt{\left(\frac{\omega_e}{k}\right)^2 + c^2}$$

b) since $\left(\dfrac{\omega_e}{k}\right)^2 > \emptyset$ for all k, $\left(\dfrac{\omega_e}{k}\right)^2 + c^2 > c^2$ for all k.

Therefore $V_{phase} > c$ for all k.

c) see attached plot.

d) $V_{group} = \dfrac{\partial \omega}{\partial k} = \dfrac{\frac{1}{2}(2c^2k)}{\sqrt{\omega_e^2 + (ck)^2}} = \dfrac{c^2k}{\sqrt{\omega_e^2 + (ck)^2}}$

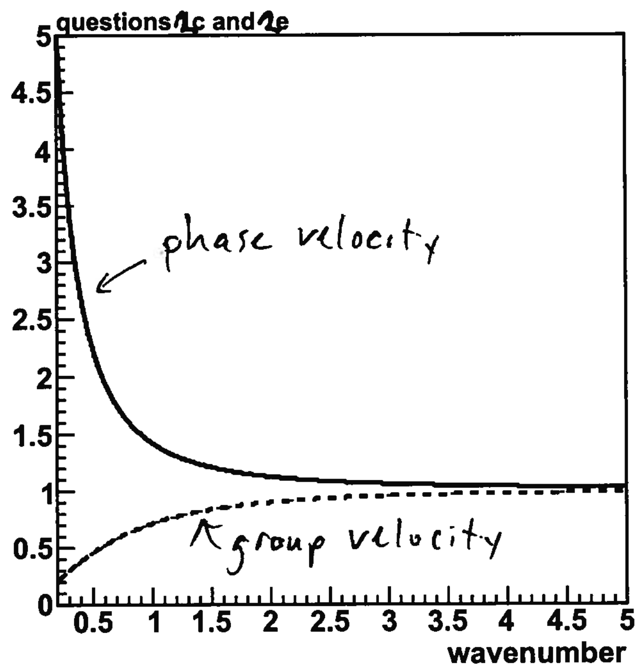
e) see attached plot.

f) $\dfrac{V_{group}}{c} = \dfrac{ck}{\sqrt{\omega_e^2 + (ck)^2}}$

since $\omega_e^2 > \emptyset$, $\sqrt{\omega_e^2 + (ck)^2} > (ck)$.

Therefore $\dfrac{ck}{\sqrt{\omega_e^2 + (ck)^2}} < 1$.

So $\dfrac{V_{group}}{c} < 1$ for all values of k.


questions 1c and 2e
phase velocity
group velocity
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
wavenumber

5

5) No, special relativity is not violated, because information is transferred at the group velocity, which is less tha the speed of light.