Phys 273 – Formula Sheet #2

$$m\ddot{x}_1 + (k + k_{12})x_1 - k_{12}x_2 = 0, \quad m\ddot{x}_2 + (k + k_{12})x_2 - k_{12}x_1 = 0 \rightarrow x_1(t) = a_1 e^{i\omega_1 t} + a_2 e^{i\omega_2 t}, \quad x_2(t) = a_1 e^{i\omega_1 t} - a_2 e^{i\omega_2 t}, \quad \omega_1 = \omega_S = \sqrt{k/m}, \quad \omega_2 = \omega_L = \sqrt{(k + 2k_{12})/m}$$

$$\vec{x}(t) = \sum_{n=1}^{2} a_n \vec{q}_n e^{i\omega_n t}, \; \vec{q}_1 = (1,1), \; \vec{q}_1 = (1,-1), \; a_i = \frac{\vec{y}_0 \cdot \vec{q}_i}{|\vec{q}_i|^2}$$

$$\ddot{y}_p + 2\omega_0^2 y_p - \omega_0^2 (y_{p+1} + y_{p-1}) = 0, \; \omega_0^2 = T/ml, \rightarrow \vec{x}(t) = \sum_{n=1}^{N} a_n \vec{q}_n e^{i\omega_n t}$$

$$\vec{q}_n = \left(\sin\left(\frac{n\pi}{N+1}\right), \sin\left(\frac{2n\pi}{N+1}\right), \ldots, \sin\left(\frac{Nn\pi}{N+1}\right) \right), \; A_{pn} = \sin\left(\frac{pn\pi}{N+1}\right), \; a_i = \frac{\vec{y}_0 \cdot \vec{q}_i}{|\vec{q}_i|^2}$$

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}, \; v = \sqrt{T/\rho}$$

$$y(x,t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) e^{i\omega_n t}, \; \omega_n = \sqrt{\frac{T}{\rho}} \frac{n\pi}{L}, \; c_n = \frac{2}{L} \int_0^L y(x, t=0) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$y(x,t) = A\sin(kx - \omega t), \; k = \frac{2\pi}{\lambda}, \; \omega = 2\pi f = \frac{2\pi}{T}, \; v = \lambda f = \frac{\omega}{k}$$

$$y(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk A(k) e^{i(kx + \omega t)}, \; A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \frac{L}{2} \delta_{nm}, \; \sum_{j=1}^{N} \sin\left(\frac{jn\pi}{N+1}\right) \sin\left(\frac{jm\pi}{N+1}\right) = \left(\frac{N+1}{2}\right) \delta_{nm}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right],$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx,\ b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L},\ c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-in\pi x/L}dx,$$

$$a_n = c_n + c_{-n},\ b_n = i(c_n - c_{-n}),\ a_0 = 2c_0$$

$$c_n = \begin{cases} \frac{1}{2}(a_{-n} + ib_{-n}), & n < 0 \\ \frac{1}{2}a_0, & n = 0 \\ \frac{1}{2}(a_n - ib_n), & n > 0 \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk A(k)e^{ikx},\ A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x)e^{-ikx}$$