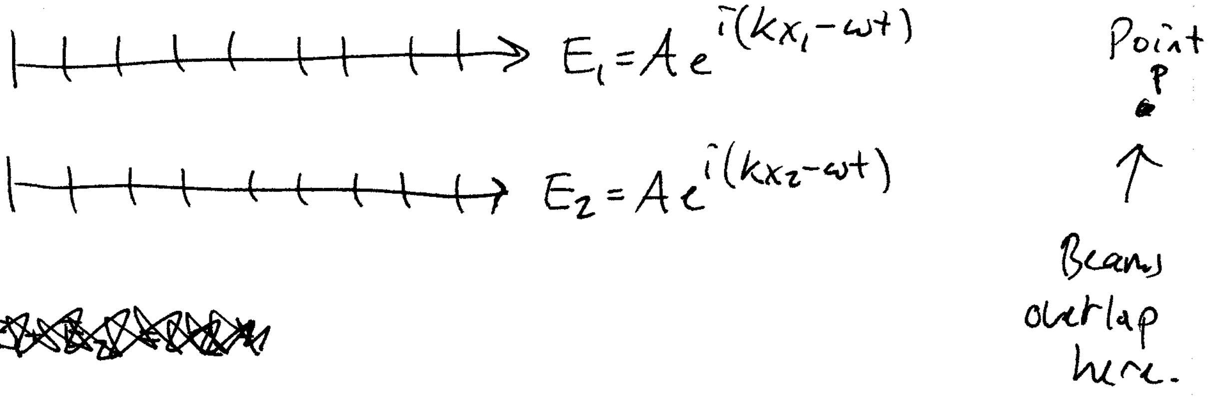
Two Beam Interference:



$$E_1 = Ae^{i(kx_1 - \omega t)}$$

$$E_2 = Ae^{i(kx_2 - \omega t)}$$

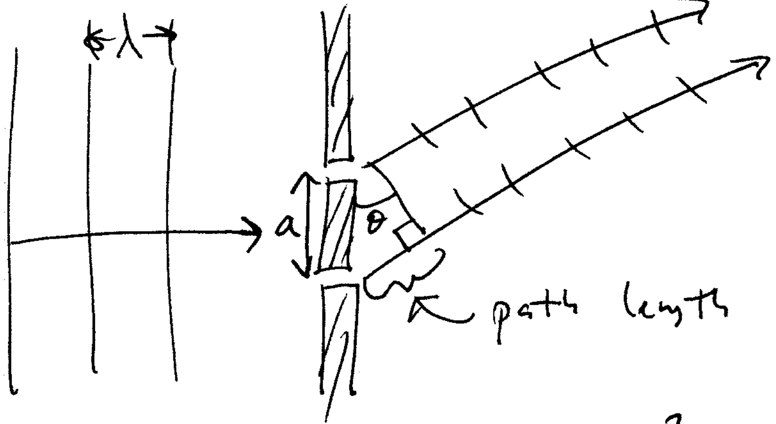
At the point of overlap,

$$I \sim 4A^2 \cos^2\left(\frac{\Delta\phi}{2}\right),$$

where $\Delta\phi$ = phase difference of the two waves at point P.

If the phase difference is due to a path-length difference, then $\underset{\text{path length difference}}{\Delta\phi = k\delta}$

Young's Double Slit experiments

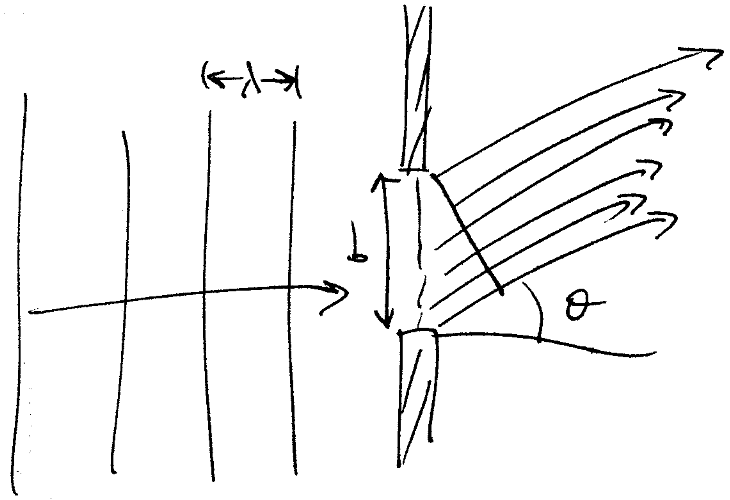


path length difference $= \delta = a \sin\theta$

$$\text{So } I \sim 4A^2 \cos^2\left(\frac{ka\sin\theta}{2}\right)$$

$$= 4A^2 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$

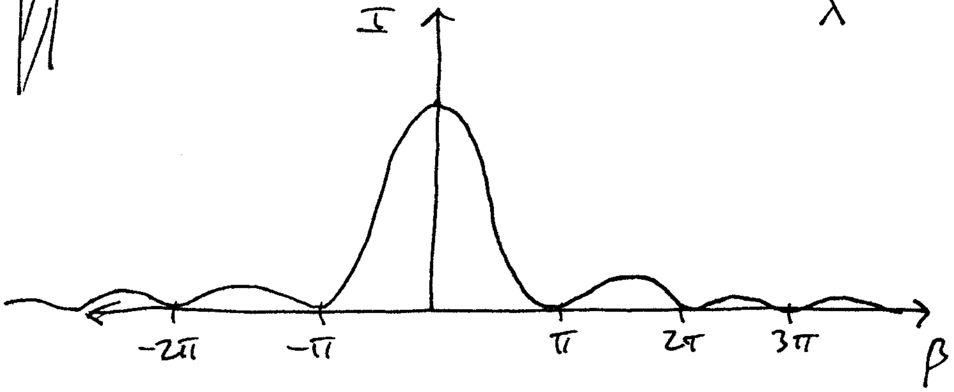
Single Slit Diffraction:



$$I \sim \frac{\sin^2\beta}{\beta^2}$$

where $\beta \equiv \frac{1}{2}kb\sin\theta$

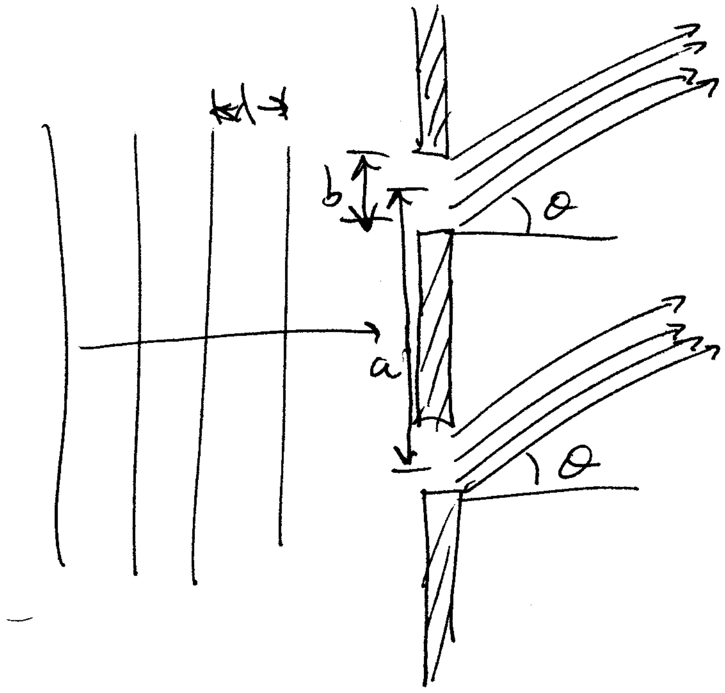
$$= \frac{\pi b \sin\theta}{\lambda}$$



Zeros occur when $\beta = m\pi$, $m = \pm 1, \pm 2, \ldots.$

or $\underline{m\lambda = b\sin\theta}$

Double Slit Diffraction:



b = slit width

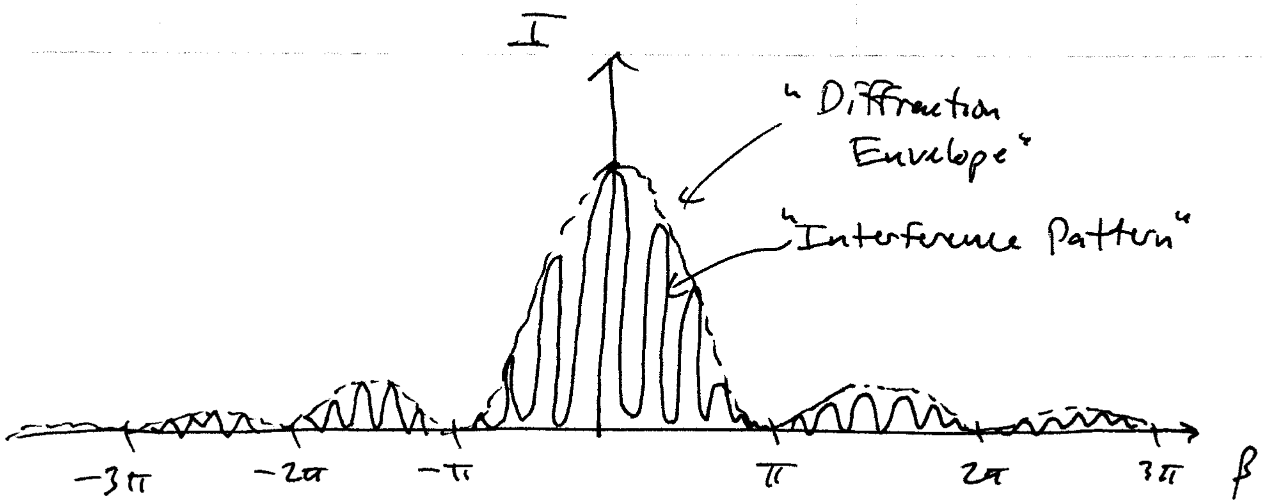
a = slit spacing

Then

$$I \sim \frac{\sin^2\beta}{\beta^2}\cos^2\alpha$$

where $\beta \equiv \frac{1}{2}kb\sin\theta$

and $\alpha \equiv \frac{1}{2}ka\sin\theta$



Maxwell's Equations & EM Waves

In integral form:

$$\oint_{\text{surface}} \vec{E}\cdot\hat{n}\,da = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$\oint_{\text{surface}} \vec{B}\cdot\hat{n}\,da = 0$$

$$\oint_{\text{curve}} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{\text{curve}} \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

In Differential Form:

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Vector Calculus:

$$\vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \text{"The Divergence"}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$$

= "The curl"

or "The circulation"

- The Divergence measures how much a vector field is spreading out.

- The curl measures how much it tends to rotate

The Fundamental Theorem of ~~[illegible]~~ Calculus:

For Divergences: $$\int_{\text{volume}} (\vec{\nabla} \cdot \vec{v})\, dV = \oint_{\text{surface}} \vec{v} \cdot \hat{n}\, da$$ "Gauss' Theorem" or "Divergence Theorem"

For Curls: $$\int_{\text{surface}} (\vec{\nabla} \times \vec{v}) \cdot \hat{n}\, da = \oint_{\text{curve}} \vec{v} \cdot d\vec{l}$$ "Stokes Theorem"

We use these two theorems to convert between the <u>integral</u> and <u>differential</u> forms of Maxwell's Equations.

Wave Equation for $\vec{E}$: $$\vec{\nabla}^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$ (in vacuum)

Component by Component: $$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial z^2}$$

For the magnetic field: $$\vec{\nabla}^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

As a consequence, $v_{phase} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/s

= speed of light

Plane wave solution:

$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}, \quad \vec{B}(z,t) = \vec{B}_0 e^{i(kz-\omega t)}$$

$\vec{E}$ & $\vec{B}$ are related:

- They are perpendicular to each other:

  $\vec{E} \cdot \vec{B} = 0$ for plane waves

- They are perpendicular to the direction of travel:

$$\left.\begin{aligned} \vec{E}_0 \cdot \hat{z} &= 0 \\ \vec{B}_0 \cdot \hat{z} &= 0 \end{aligned}\right\} \text{ for travel in the } z\text{-direction}$$

- Their magnitudes are related by

$$|\vec{E}_0| = c|\vec{B}_0|$$

Poynting Vector

For a plane wave in vacuum,

u_B = energy density in the magnetic field = u_E (energy density in the electric field)

total energy density = $u = \varepsilon_0 |\vec{E}|^2$

Poynting Vector: $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ = energy per unit area carried by the plane wave.

Also, $\vec{S}$ points in the direction of travel.

The time average of $\vec{S}$ is the Intensity:

$$\langle \vec{S} \rangle = \text{Intensity} = I = \tfrac{1}{2} c \varepsilon_0 |\vec{E}|^2$$

Dielectrics

For linear dielectric materials we can replace $\varepsilon_0 \to \varepsilon$ and $\mu_0 \to \mu$.

Then the wave equation in the dielectric is

$$\nabla^2 \vec{E} = \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

The phase velocity is

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}}$$

And the index of refraction is defined to be

$$n \equiv \frac{c}{v_p} \quad \leftarrow \text{speed of light in vacuum}$$

$$= \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

$$\approx \sqrt{\frac{\varepsilon}{\varepsilon_0}} \quad \text{if} \quad \mu \approx \mu_0 .$$

$$n > 1 .$$

The Poynting Vector in the dielectric is

$$\vec{S} = \frac{1}{\mu}\vec{E}\times\vec{B}$$

Reflection and Transmission at a Dielectric Boundary (at normal incidence).

$$\underset{\substack{\uparrow \\ \text{reflected electric field}}}{E_{0R}} = \left(\frac{Z_2 - Z_1}{Z_1 + Z_2}\right) \underset{\substack{\uparrow \\ \text{incident electric field}}}{E_{0I}}$$

and

$$E_{0T} = \left(\frac{2Z_1}{Z_1 + Z_2}\right) E_{0I}$$

transmitted electric field

where $Z \equiv \mu v_p = \sqrt{\frac{\mu}{\varepsilon}}$ for EM waves in dielectrics

For free space, $Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\,\Omega$.

We can write the reflection coefficient in terms of the index of refraction.

$$E_{0R} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_{0I}$$

Then

$$I_{\text{reflected}} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) I_{\text{incident}}$$