**Physics 273 - Homework #4**

1) a ) Suppose we construct a parallel plate capacitor using two square copper plates separated by a distance of 0.0025 cm. (Maybe we use a thin piece of paper to separate the plates.) Each plate is 10 cm by 10 cm square. Look up the expression for an ideal parallel plate capacitor and calculate the expected capacitance for our device. You may assume that the material in the gap between the plates is vacuum. Please give your answer in MKS units for capacitance.

b) Now we construct an inductor by wrapping a copper wire around a plastic soda bottle. The bottle is 7 cm in diameter and 15 cm long. We use 22 gauge wire (0.064 cm diameter), which allows us to fit 234 tightly packed loops on the 15 cm length of the bottle. Look up the expression for an ideal solenoid and calculate the expected self-inductance for our device. As in part (a), assume that the interior of the bottle has the electrical properties of vacuum. Please give your answer in MKS units for inductance.

c) We charge up the capacitor by connecting it to a 9 V battery. How much charge is on each plate of the capacitor?

d) How much energy is stored in the capacitor after it is charged?

e) We now connect the capacitor and the inductor to form an LC oscillator. What is its natural frequency?

f) Copper is not a perfect conductor, so our inductor has some resistance as well as inductance. Therefore a better model for our circuit would be an RLC circuit, where the resistor, capacitor, and inductor are all connected in series. At room temperature the bulk resistivity (volume resistivity) of copper is 1.72 x $10^{-8}$ Ohm-meters. Use this number and the dimensions of the copper wire to calculate the expected resistance of our inductor. Please give your answer in Ohms. (Look up the definition of bulk resistivity if necessary.)

g) How long will it take for the energy stored in the LC circuit to decrease to 50% of its initial value?

h) What is the Q value of our circuit?

i) Suppose we want to use our circuit to detect the AM radio signal from WTOP, which broadcasts at a frequency of f = 1500 kHz. We will need to change the geometry or material of our capacitor and/or inductor to make our circuit resonate at the WTOP frequency. Suppose we decide to modify the capacitor by changing the gap between the plates. What distance should we use for the gap?

2) **Numerical solution of a forced harmonic oscillator with damping.** The equation of motion for the forced harmonic oscillator with damping is

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = F(t)/m$$

where $F(t)$ is the forcing function. (As with the damped harmonic oscillator, ($\gamma = b/m$), ($b$) is the drag coefficient, and ($\omega_0^2 = k/m$)). Suppose that the forcing function has the form of a cosine function:

$$F(t) = F_0 \cos(\omega_f t)$$

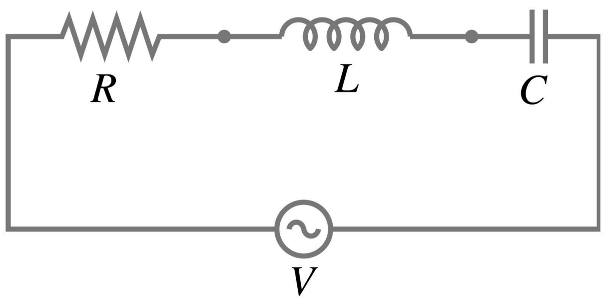
where ($F_0$) has units of Newtons, and ($\omega_f$) is the forcing frequency.

Copy your numerical solution to the damped harmonic oscillator (Homework #3, problem #4) and modify it by adding in the effects of the external force. Use the following parameters in your numerical solution: $k = 10$ N/meter, $m = 1$ kg, $b = 0.3$ N/(meter/second), $F_0 = 0.5$ Newtons. Use a time step of 0.01 seconds, and calculate for 4000 time steps (a total of 40 seconds). For the initial conditions, assume that the oscillator starts from rest at its equilibrium position ($x_0 = 0.0$ and $v_0 = 0.0$). (The last parameter in the problem is the forcing frequency ($\omega_f$), which we will vary in the following questions.)

a) Calculate by hand the expected natural frequency ($\omega_0$) for this oscillator.
b) Calculate by hand the expected damped frequency ($\omega_d$). (This is the frequency of the oscillator in the absence of a forcing function, but in the presence of the damping force.)
c) Set the forcing frequency to $\omega_f = 1$ Hz, and print out a plot of the oscillator position as a function of time for the first 40 seconds. Please set the y-axis limits of your plot to +/- 0.60 meters. Also, please mark the region where the transient solution has a noticeable effect, and mark the region where the transient solution does not have a noticeable effect.
d) Now set the forcing frequency to $\omega_f = 0.1$ Hz. In your solution you should see clearly the effect of the damped frequency ($\omega_d$). Measure the value of the damped frequency as observed in your solution, and compare it to the expected value from part (b).
e) Now we will examine the steady-state part of the solution and how it depends on the forcing frequency. For this question, set the y-axis limits on your plot to be +/- 0.60 meters, and do not let those limits change as you alter the forcing frequency. First set the forcing frequency to $\omega_f = 1$ Hz and measure the amplitude of the steady-state solution. Hint: if you are using Excel, you can use the MAX() function to automatically "observe" this amplitude. For example, MAX(D3011:D4011) returns the maximum value of the cells D3011 through D4011.
f) Step the forcing frequency from 1.0 Hz to 5.0 Hz in 0.1 Hz steps (a total of 41 different forcing frequencies). At each step, record the amplitude of the steady-state part of the solution. Make a plot of this amplitude as a function of the forcing frequency, and mark on your plot the value of the natural frequency which you calculated in part (a).
g) Among those forcing frequencies that you tried in part (g), which gives the largest amplitude for the steady state solution? Print out a plot of the position of the oscillator as a function of time for this particular forcing frequency.

3) **Series RLC circuit.**

Consider a series RLC circuit driven by a voltage source:




By considering the phasor diagram for the voltages in this circuit, we found the following expression for the circuit impedance:

$$|Z_{series}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \ \omega = \text{driving frequency.}$$

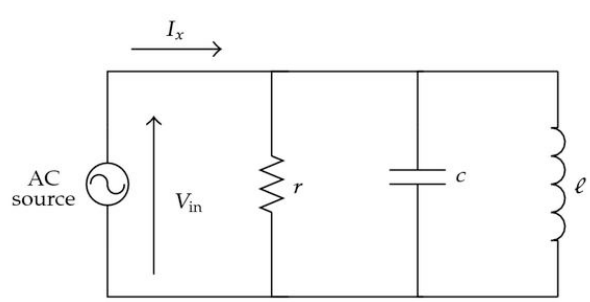
a) Show that you can deduce this expression for the impedance without a phasor diagram simply by using the rules for combining impedances in series.

b) Suppose that the magnitude of the voltage source is one volt, the inductance is 1 milli-Henry, and the capacitance is one milli-Farad. Use a plotting program to make a three plots of the **peak current as a function of driving frequency** for three resistance values: R = 0.025 Ohms, R = 0.1 Ohms, and R = 0.4 Ohms. Please plot the peak current value for driving frequencies (ω) ranging from 2 Hz to 2,000 Hz, and please put the three curves on the same plot, so we can compare your curves by eye.

c) Please label your three curves according to the resistance value of each. Qualitatively speaking, what is the difference between the three curves?

d) Does this current in this circuit have a resonance behavior? If so, what resistance value gives the highest Q factor, and which resistance value gives the smallest Q factor?

4) **Parallel RLC circuit.** Here is a driven RLC circuit where the circuit elements are connected in parallel:




a) How many independent voltages are there in this circuit?

b) Draw a phasor diagram for this circuit. Start by drawing the voltage phasor for the voltage source, and then add three current phasors, one each for the resistor, capacitor, and inductor.

c) There is a fourth current phasor in the circuit, the current delivered by the voltage source. We can determine its relationship to the other three current phasors using Kirchoff's current sum rule. Write down the current sum rule for this circuit as an equation, and draw it as a phasor diagram.

d) Using the rules for combining impedances in parallel, show that the impedance for this circuit can be written as

$$Z_{para} = \frac{R\omega L}{\omega L + i\left(\omega^2 RLC - R\right)},$$ where ω is the driving frequency.

e) Prove the following identity for complex numbers: If $z = \frac{a}{b + ic}$, then $|z| = \frac{a}{\sqrt{b^2 + c^2}}$.

f) Use the identity from part (e) to find an expression for the magnitude of $Z_{para}$ as a function of the driving frequency, R, L, and C.

g) Suppose that the magnitude of the voltage source is one volt, the inductance is 1 milli-Henry, and the capacitance is one milli-Farad. Use a plotting program to make a three plots of the **peak current as a function of driving frequency** for three resistance values: R = 0.1 Ohms, R = 1.0 Ohms, and R = 10.0 Ohms. Please plot the peak current value for driving frequencies (ω) ranging from 2 Hz to 20,000 Hz, and please put the three curves on the same plot, so we can compare your curves by eye.

Note: In question (1b), I asked you to plot up to ω = 2,000 Hz, whereas in this question I have asked you to plot up to 20,000 Hz.