Phys 273 – Formula Sheet #1

$F = -kx$, $U(x) = \frac{1}{2}kx^2$, $\ddot{x} + \omega_0^2 x = 0$, $\omega_0 = \sqrt{k/m}$, $x(t) = Ae^{i(\omega t+\delta)}$

$U(x_0 + a) = U(x_0) + U'(x_0)a + \frac{1}{2}U''(x_0)a^2 + \ldots,$

$\omega_0 = \sqrt{g/l}$

$z = x + iy = Ae^{i\vartheta}$, $A = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$, $x = A\cos(\theta)$, $y = A\sin(\theta)$

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}e^{i\omega t}$, $\gamma = b/m$, $F_{drag} = -bv = -b\dot{x}$,

$x(t) = A(\omega)e^{i(\omega t+\delta(\omega))}$, $A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}$, $\delta(\omega) = -\tan^{-1}\left[\frac{\omega\gamma}{(\omega_0^2 - \omega^2)}\right]$

$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$, $x(t) = Ae^{-\gamma t/2}e^{i(\omega_d t+\delta)}$ $\omega_d = \sqrt{\omega_0^2 - \gamma^2/4}$

$KE = \frac{1}{2}m\dot{x}^2$, $U = \frac{1}{2}kx^2$,

$U_E = \int \frac{1}{2}\varepsilon_0 \left|\vec{E}^2\right| dV = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$, $U_B = \int \frac{1}{2\mu_0}\left|\vec{B}^2\right| dV = \frac{1}{2}LI^2$

$E(t) = E_0 e^{-\gamma t}$, $Q \equiv \omega_0/\gamma$

$V_R = IR$, $Vc = \frac{1}{C}Q$, $V_L = L\frac{dI}{dt}$

$Z_R = R$, $Z_L = i\omega L$, $Z_C = \frac{-i}{\omega C}$, $Z_{series} = Z_1 + Z_2 + \ldots$, $Z_{parallel} = \frac{1}{Z_1^{-1} + Z_2^{-1} + \ldots}$

$\vec{V} = \vec{I}Z$

$$\begin{aligned} m\ddot{x}_1 + (k + k_{12})x_1 - k_{12}x_2 &= 0 \\ m\ddot{x}_2 + (k + k_{12})x_2 - k_{12}x_1 &= 0 \end{aligned}, \rightarrow \begin{aligned} x_1(t) &= a_1 e^{i\omega_1 t} + a_2 e^{i\omega_2 t} \\ x_2(t) &= a_1 e^{i\omega_1 t} - a_2 e^{i\omega_2 t} \end{aligned}, \quad \begin{aligned} \omega_1 &= \omega_S = \sqrt{k/m} \\ \omega_2 &= \omega_L = \sqrt{(k + 2k_{12})/m} \end{aligned}$$

$\vec{x}(t) = \sum_{n=1}^{2} c_n \vec{q}_n e^{i\omega_n t}$, $\vec{q}_1 = (1,1)$, $\vec{q}_1 = (1,-1)$, $c_n \equiv a_n + ib_n$, $a_n = \frac{\vec{x}_0 \cdot \vec{q}_n}{|\vec{q}_n|^2}$, $b_n = \frac{-\vec{v}_0 \cdot \vec{q}_n}{\omega_n |\vec{q}_n|^2}$