This review covers the material since exam 2.

Energy Transport by travelling wave on a string:

Recall that Z = mechanical impedance of the string

$$= \frac{T}{v_p} \quad \begin{matrix} \leftarrow \text{tension} \\ \leftarrow \text{phase velocity} \end{matrix}$$

$$= \rho v_p$$

ρ ← mass density

Then Energy transmitted per second = Power $= \frac{1}{2} Z \omega^2 A^2$

Z ← impedance, ω ← frequency, A ← amplitude

Transmission Lines

L_0 = inductance per unit length
C_0 = capacitance per unit length

Then $V_{phase} = \frac{1}{\sqrt{L_0 C_0}}$

Z_0 = "characteristic impedance" $\equiv \frac{V_0}{I_0}$

and $Z_0 = \sqrt{\frac{L_0}{C_0}}$ for a transmission line

Reflection & Transmission of Voltage waves:

If Z_L = load impedance

and Z_0 = characteristic impedance of the line, then $$\frac{V_-}{V_+} = \frac{\text{Reflected Amplitude}}{\text{Incoming Amplitude}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\frac{V_L}{V_+} = \frac{\text{Transmitted Amplitude}}{\text{Incoming Amplitude}} = \frac{2 Z_L}{Z_L + Z_0}$$

Reflection & Transmission of Current Waves:

$$\frac{I_-}{I_+} = \frac{\text{Reflected Amplitude}}{\text{Incoming Amplitude}} = \frac{Z_0 - Z_L}{Z_0 + Z_L}$$

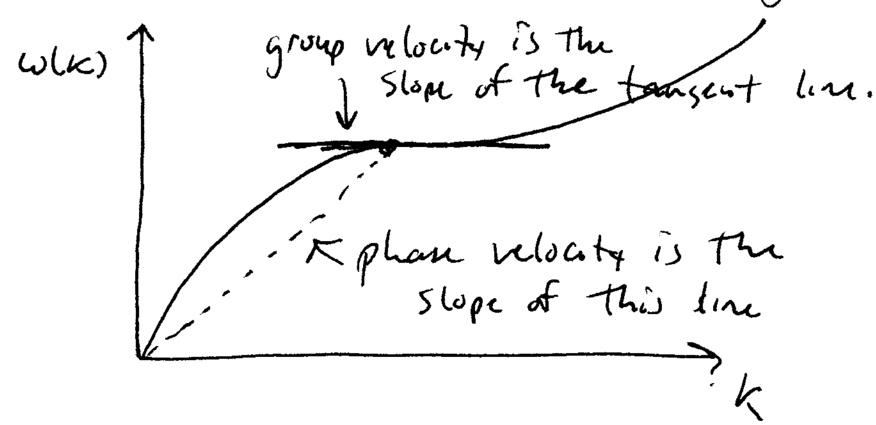
$$\frac{I_L}{I_+} = \frac{\text{Transmitted Amplitude}}{\text{Incoming Amplitude}} = \frac{2 Z_0}{Z_L + Z_0}$$

Dispersion

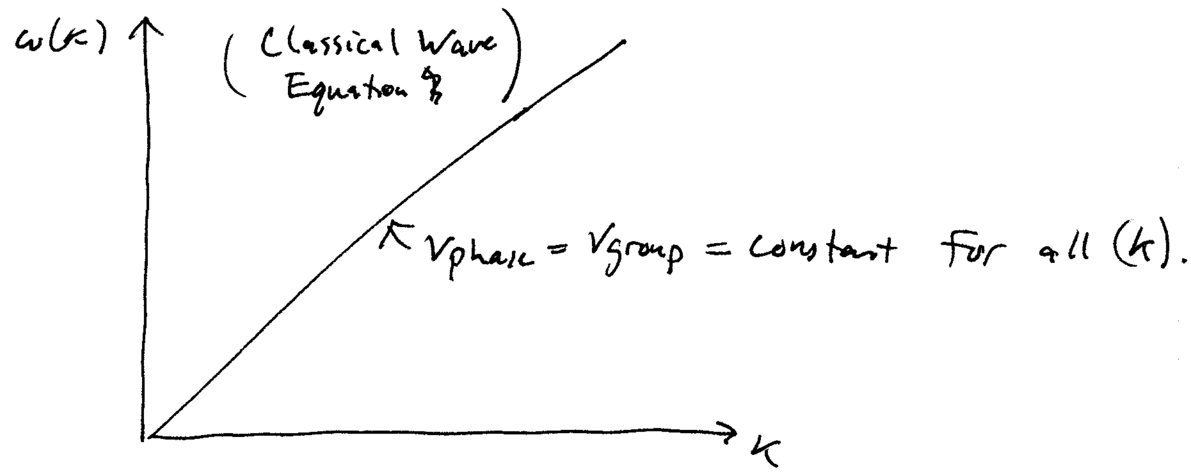
$$\text{phase velocity} = \frac{\omega(k)}{k}$$

$$\text{group velocity} = \frac{\partial \omega(k)}{\partial k} \quad \text{or} \quad \frac{d\omega(k)}{dk}$$

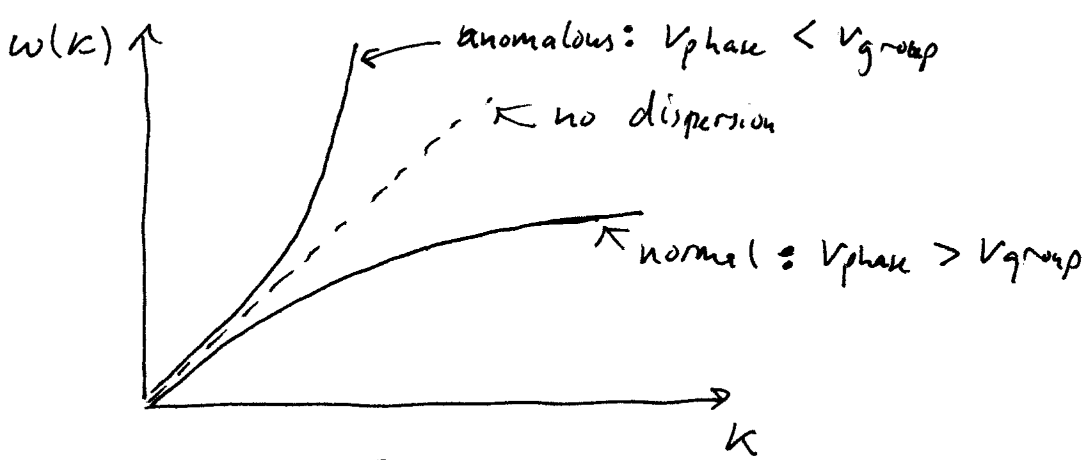
If the phase velocity depends upon k, then pulses will disperse. Information travels at the speed of the pulse envelope, which is the group velocity



Systems described by the classical wave equation must have no dispersion:



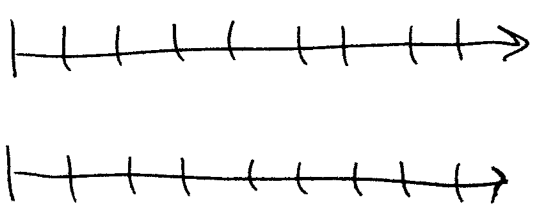
Other cases are "normal" and "anomalous"



In any medium ~~where~~ with non-zero dispersion, pulses will tend to dissipate as they propagate forward. This is because the component travelling waves are all travelling at their own velocity, so they will get out of step with each other.

In a non-dispersive medium (classical wave equation), all the component travelling waves advance together, and the pulse will maintain its shape indefinitely.

Two Beam Interference:



$E_1 = Ae^{i(kx_1 - \omega t)}$

$E_2 = Ae^{i(kx_2 - \omega t)}$

Point P

Beams overlap here.

At the point of overlap,

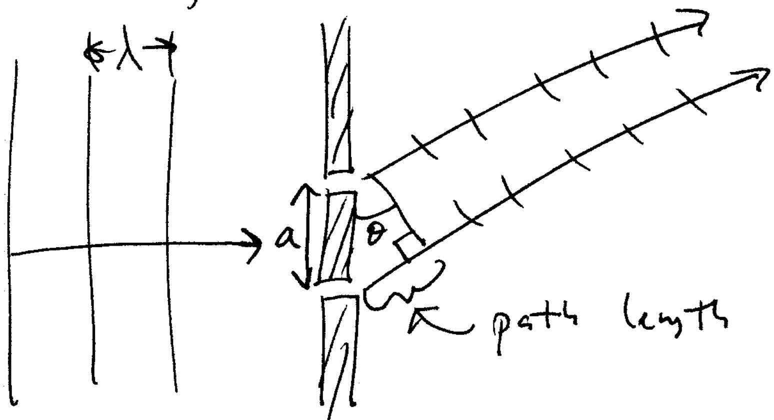
$$I \sim 4A^2 \cos^2\left(\frac{\Delta\phi}{2}\right),$$

where $\Delta\phi$ = phase difference of the two waves at point P.

If the phase difference is due to a path-length difference, then $\Delta\phi = k\delta$

δ ← path length difference

Young's Double Slit experiments

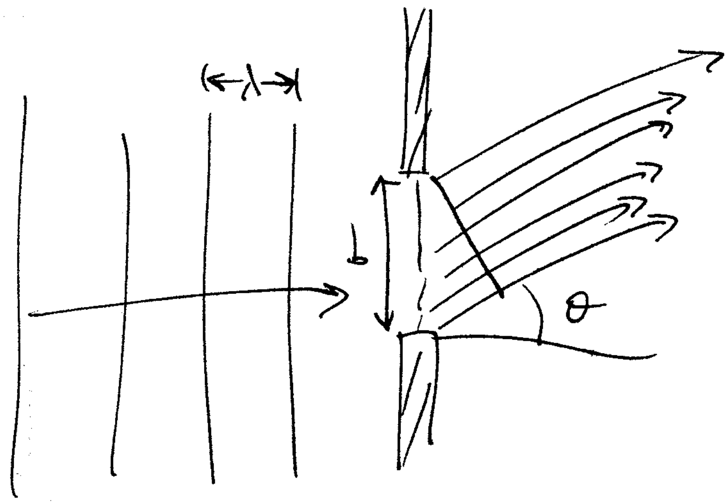


path length difference $= \delta = a\sin\theta$

$$\text{So } I \sim 4A^2 \cos^2\left(\frac{ka\sin\theta}{2}\right)$$

$$= 4A^2 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$

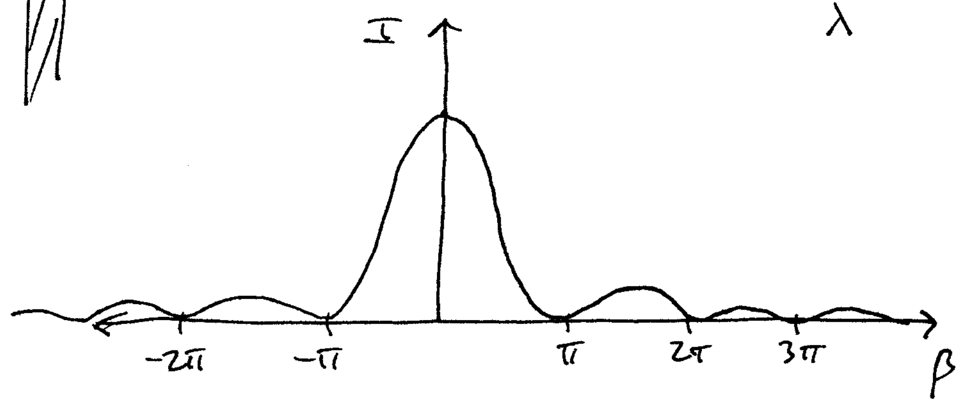
Single Slit Diffraction:



$$I \sim \frac{\sin^2\beta}{\beta^2}$$

where $\beta \equiv \frac{1}{2} k b \sin\theta$

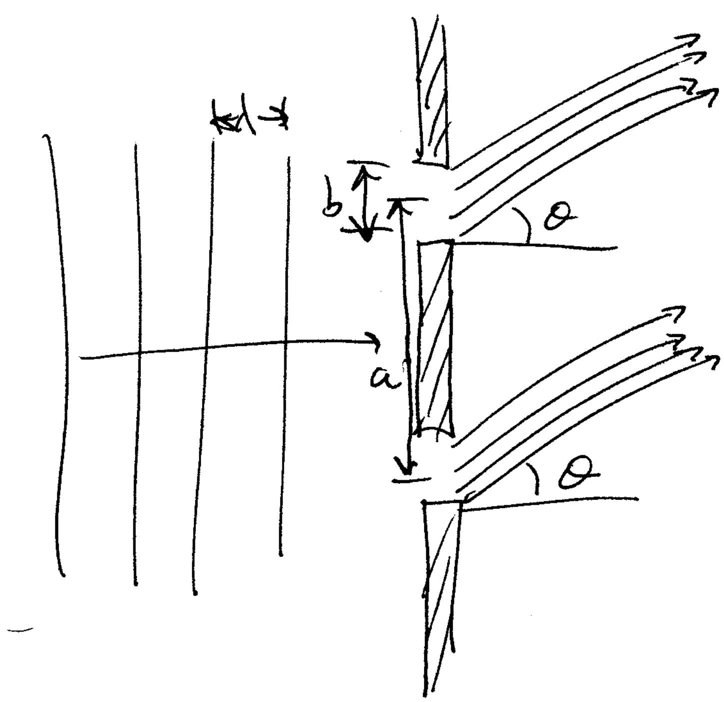
$$= \frac{\pi b \sin\theta}{\lambda}$$



Zeros occur when $\beta = m\pi$, $m = \pm 1, \pm 2, \ldots$

or $\underline{m\lambda = b\sin\theta}$

Double Slit Diffraction:



b = slit width

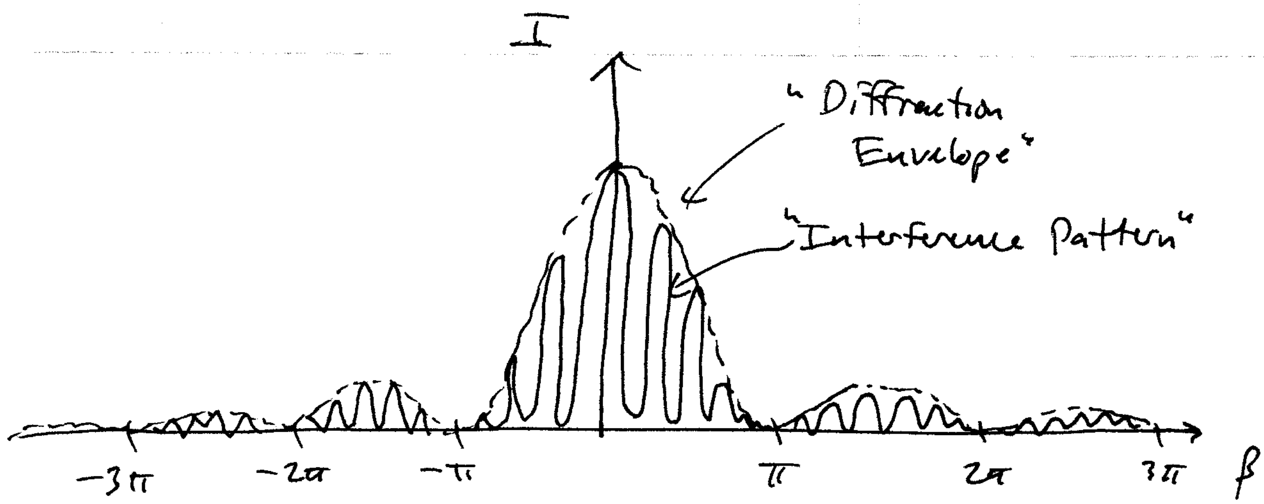
a = slit spacing

Then

$$I \sim \frac{\sin^2\beta}{\beta^2}\cos^2\alpha$$

where $\beta \equiv \frac{1}{2} k b \sin\theta$

and $\alpha \equiv \frac{1}{2} k a \sin\theta$



Maxwell's Equations & EM Waves

In integral form:

$$\oint_{\text{surface}} \vec{E} \cdot \hat{n}\, da = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\oint_{\text{surface}} \vec{B} \cdot \hat{n}\, da = \phi$$

$$\oint_{\text{curve}} \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint_{\text{curve}} \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In Differential form:

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla} \cdot \vec{B} = \phi$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Vector Calculus:

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \text{"The Divergence"}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$$

= "The curl"

or "The circulation"

- The Divergence measures how much a vector field is spreading out.
- The curl measures how much it tends to rotate.

The Fundamental Theorem of ~~Calculus~~ Calculus:

For Divergences: $\int_{\text{volume}} (\vec{\nabla} \cdot \vec{v})\, dV = \oint_{\text{surface}} \vec{v} \cdot \hat{n}\, da$ "Gauss' Theorem" or "Divergence Theorem"

For Curls: $\int_{\text{surface}} (\vec{\nabla} \times \vec{v}) \cdot \hat{n}\, da = \oint_{\text{curve}} \vec{v} \cdot d\vec{l}$ "Stokes Theorem"

We use these two theorems to convert between the <u>integral</u> and <u>differential</u> forms of Maxwell's Equations.

Wave Equation for $\vec{E}$: $\vec{\nabla}^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ (in vacuum)

Component by Component:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial z^2}$$

For the magnetic Field: $\vec{\nabla}^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

As a consequence, $v_{phase} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

= speed of light

~~Also~~ Plane wave solution:

$$\vec{E}(z,t) = \vec{E}_0 e^{i(kz-\omega t)}, \quad \vec{B}(z,t) = \vec{B}_0 e^{i(kz-\omega t)}$$

$\vec{E}$ & $\vec{B}$ are related:

- They are perpendicular to each other:

$$\vec{E} \cdot \vec{B} = \emptyset \quad \text{for plane waves}$$

- They are perpendicular to the direction of travel:

$$\left.\begin{array}{l} \vec{E}_0 \cdot \hat{z} = \emptyset \\ \vec{B}_0 \cdot \hat{z} = \emptyset \end{array}\right\} \text{for travel in the } z\text{-direction}$$

- Their magnitudes are related by

$$|\vec{E}_0| = c|\vec{B}_0|$$